ANALYSIS AND MODELING OF FREQUENCY-DEPENDENT RESISTANCE AND INDUCTANCE OF SINGLE CMOS INTERCONNECTS

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Abstract

A simple, efficient CAD-oriented analytic modeling approach to calculate frequency-dependent distributed inductance and associated distributed series resistance per-unit-length of single on-chip interconnects on a lossy silicon substrate is presented. The closed-form formulas for the frequency-dependent series impedance parameters are obtained using vector magnetic potential equation and a complex image technique. The proposed frequency-dependent inductance $L(\omega)$ and resistance $R(\omega)$ per-unit-length formulas are shown to be in good agreement with the electromagnetic solutions.

Keywords: Analytic formulas, on-chip interconnects, silicon substrate, eddy currents, complex image technique.

1. Introduction

As technology advances into the very deep-submicron era, interconnection delay dominates overall circuit performance and noise becomes more serious than before. Therefore, accurately predicting the interconnection delay and noise becomes a major challenge in high performance designs. For deep-submicron, high performance circuits, ignoring series line impedance effects may incur a large amount of error, since an RC model as compared to an RLC model may create errors of up to 30 % in the total propagation delay of a repeater system [1], and in some worst cases the noise coupling due to C and L may reach around 55 % of the supply voltage [3]. As technology improves and die size increases, short rise/fall times of signals and long wires make inductance effects much more significant than before [4]. Moreover, for substrates with higher conductivity (silicon), such as in CMOS technology, the inductance becomes frequency-dependent due to the variation of magnetic flux

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penetration into the silicon substrate as a function of frequency, which is called the substrate skin effect [5, 7].

There has been much interest in recent years in determining the frequency-dependent characteristics of on-chip interconnects and passive components on silicon oxide-silicon substrate. The broad-band transmission line behaviour of interconnect lines on insulator-semiconductor substrate has been studied by rigorous full-wave electromagnetic solutions [8 – 11] and more recently by quasi-TEM EM computation techniques [6, 11, 12, 18, 19] including of equivalent circuit models [6, 12].

Although metal-insulator-semiconductor interconnects structures has been studied from various point of view, as mentioned above, there still exists a bottleneck in the whole simulation process or design cycle for on-chip interconnects analysis. In a word, the bottleneck of whole process in determining the characteristics of on-chip interconnects is still the time consuming EM simulation. It is then highly desirable to have analytic or closed-form model for frequency-dependent line parameters of on-chip interconnects that are suitable for CAD and thus can significantly speed up the simulation.

In this paper, a complex image method in cojunction with vector magnetic potential equation is applied to calculate the frequency dependent inductance and the associated series resistance of single interconnect on lossy silicon substrate (CMOS technology). To illustrate the accuracy of the model, the frequency-dependent series impedance parameters (inductance and resistance) of single on-chip interconnects on lossy silicon substrate were computed using proposed analytic formulas, and compared with the solution by electromagnetic full wave solutions and CAD-oriented equivalent-circuit modeling approach.

2. Extraction of Series Impedance Parameters

Single interconnect line (microstrip) may be the simplest on-chip interconnects configuration but has fundamental importance. By examining the single line case (see Fig. 1), different mechanisms that contribute to the frequency-dependent line parameters can be carefully studied and accordingly the methodology of developing closed-form model can be formulated.

The determination of series impedance parameters R(ω) and L(ω) should take into account the effects due to semiconducting substrate, finite thickness and finite conductivities of conducting strips. However, the later effects have been extensively studied and easily available in terms of closed-form expressions for conductor loss and conductor skin effect. On the other hand, the effects due to the silicon substrate are difficult to describe in terms of closed-form model. The contributions to the overall characteristics of single on-chip interconnects due to lossy silicon substrate (CMOS technology) and no ideal conductor are approximately additive. This allows us to concentrate the more complicated silicon substrate loss effect.

For on-chip interconnects, the silicon substrates with varying doping levels have varying resistivity. Particularly for substrate with low resistivity, the time-varying magnetic fields in the substrate give rise to the frequency-dependent fields in the substrate. This results in frequency-dependent horizontal currents (eddy currents) in the lossy substrate. At higher frequencies the presence of these substrate currents can lead to a significant reduction in series inductance and substantial increases in series resistance or loss. The evaluation of per-unit-length series impedance Z_s' can be obtained by considering the complex equivalent inductance [14, 15]

$$L_e = L(\omega) + R(\omega)/j\omega = \Phi / I = \oint_c \mathbf{A} \cdot \mathbf{dl} / I \tag{1}$$

where I is the total current on the conductor, Φ is the magnetic flux linkage associated with the interconnects, and **A** is the vector magnetic potential.

Results obtained from the full-wave analysis [5, 7, 11] have shown that the influence of the finite substrate thickness t_{si} can be neglected for practical dimensions (i.e. $t_{si} >> w$, $t_{si} >> t_m$, and $t_{si} >> t_{ox}$). The silicon substrate is therefore assumed to be infinitely thick in the following derivation.

To develope an expression for series impedance Z'_s the given structure (see Fig. 1a) can be regarded as a system of inductively coupled transmission line with the silicon substrate acting as a return line as shown in Fig. 1b.

If the quasi-TEM wave is considered, the magnetic vector potential **A** determinated as

$$\mathbf{H} = \frac{1}{\mu} rot\ \mathbf{A} \tag{2}$$

has only a z-component and satisfies the magnetic vector potential equation

$$\nabla^2 A_i - j\omega\mu_i\sigma_i A_i = -\mu_0 \delta(x-x')\delta(y-y')\,. \tag{3}$$

subject to corresponding boundary conditions at the interface between the layers in structure.

The quasi-static equation given in (3) can be accurately solved, for example, by a modified spectral domain approach to obtain the frequency-dependent line impedance parameters of the on-chip interconnects [5, 7, 11]. The solution of the (3) for the magnetic potential takes into account the complicated two-dimensional frequency-dependent distribution of eddy current in the lossy silicon substrate.

For a substrates with zero resistivity (perfectly conducting material), the substrate currents would be limited to the surface of the substrate and, hence, could be represented by an equivalent image strip current below the surface.

In this paper the concept of an image current can be extended to substrates with finite resistivity by applying a complex image theory [17, 19] as described below.

The complex image method [13, 17] is valid for lossy silicon substrates if the skin depth is much smaller than the substrate height.

Conceptually, the essence of the complex image is to replace the finitely conducting plane by a perfectly conducting ground locating at a complex depth h_{com} [13, 14]. The location of this complex image depth can be derived by several methods, one of which is to equate the z-directed wave impedance for normal incidence at the surface (silicon substrate). The derivation procedure is illustarated in Fig. 2. Fig. 2a shows the configuration with air and the semi-infinite silicon conducting medium. Fig. 2b shows the equivalent configuration with air filled but bounded by a perfectly conducting ground plane at the complex depth of h_{com}.

For case of Fig. 2a, the wave impedance for TE mode is [14]

$$Z_a = \frac{\eta_1}{\sqrt{1-\frac{\gamma_0^2}{\gamma_1^2}}} \tag{4}$$

where

$$\gamma_0^2 = -\omega^2\mu_0\varepsilon_0 \tag{5}$$

$$\gamma_1^2 = -\omega^2 \mu_0 \left(\varepsilon_{si} - j\frac{\sigma}{\omega} \right) \tag{6}$$

and

$$\eta_1 = \sqrt{\frac{\mu_0}{\varepsilon_{si} - j\frac{\sigma}{\omega}}} \, . \tag{7}$$

For the case of Fig. 2b, since the perfect ground plane exists at the distance of h_{com}, the wave impedance for TE mode can be determined by transmission line theory as [14]

$$Z_b = \eta_0 \tanh(\gamma_0 h_{com}) \tag{8}$$

where $\eta_0 = \sqrt{\mu_0 / \varepsilon_0}$. When $|\gamma_0 h_{com}|$ is small enough, (8) can be simplified as

$$Z_b \approx \eta_0 \gamma_0 h_{com} \, . \tag{9}$$

Equating (4) and (9) results in

$$h_{com} = \frac{\eta_1}{\eta_0 \gamma_0 \sqrt{1 - \frac{\eta_0^2}{\eta_1^2}}}$$

$$= \frac{1}{\gamma_1 \sqrt{1 - \frac{\eta_0^2}{\eta_1^2}}}$$

$$= \frac{1}{\sqrt{\gamma_1^2 - \gamma_0^2}}$$

$$= \frac{1}{\sqrt{j\omega\mu_0\sigma}} \, . \tag{10}$$

Noting that the skin depth is defined as $\delta = 1/\sqrt{\pi f \mu_0 \sigma}$ and $\sqrt{j} = (1+j)/\sqrt{2}$, (eq. (10) can be rewritten as $h_{com} = \delta(1-j)/2$. Correspondingly, the image depth is

$$d_{comTE} = 2h_{com} = (1-j)\delta \, . \tag{11}$$

The wave impedance for TM mode in case of structure from Fig. 2a is [14]

$$Z_a = \eta_1 \sqrt{1 - \frac{\gamma_0^2}{\gamma_1^2}} \, . \tag{12}$$

Then, the image depth for TM mode can be obtained as

$$d_{comTM} = \frac{2}{\gamma_1}\sqrt{1-\frac{\gamma_0^2}{\gamma_1^2}}\,. \tag{13}$$

For normal incidence, or if $\left|\frac{\gamma_1^2}{\gamma_0^2}\right|$ >> [15, 16], (11) and (13) are shown to be the same as

$$d_{comTE} = d_{comTM} = d_{com} = \frac{2}{\gamma_1}\,. \tag{14}$$

When $\sigma >> \omega\varepsilon_0\, \varepsilon_{rsi}$, which is always satisfied in our case, $\gamma_1 \approx \sqrt{j\omega\mu_0\sigma}$. Therefore, in terms of skin depth we defined earlier, d_{com} in (14) can be rewritten as

$$d_{com} = (1-j)\delta\,. \tag{15}$$

The effect of the lossy silicon substrate can be represented in terms of a real image current line located at complex distance $d_{com} = (1\text{-}j)\delta(\omega)$ below the surface of the lossy silicon substrate. Alternatively, an virtual ground plane can be placed at distance $d_{com}/2$ below the surface to represent the frequency-dependent eddy currents in the lossy silicon substrate.
Using the concept of the virtual ground plane, the single interconnect strip on a silicon oxide silicon substrate can be replaced by a microstrip with lossless substrate of complex effective height, as illustrated in Fig. 3. The complex effective height for the single interconnects on lossy silicon substrate is given by

$$t_{eff} = t_{ox} + \frac{1-j}{2}\delta \tag{16}$$

where t_{ox} is the oxide thickness and δ the skin depth of the bulk silicon.

Applying the concept of the virtual ground plane, the frequency-dependent series inductance and resistance parameters of a single interconnects on a oxide-silicon substrate are determined as

$$L(\omega) = \mathrm{Re}\{L_{cf}\,(h = t_{eff}\,)\} \tag{17}$$

and

$$R(\omega) = -\omega\,\mathrm{Im}\{L_{cf}\,(h = t_{eff}\,)\} \tag{18}$$

where L_{cf} represents closed-form expressions for the per-unit-length inductance of a single strip conductor with a lossless substrate of height h.

For a single strip conductor with a lossless substrate of height h, in [18] very accurate closed-form expressions for per-unit-length inductance L_{cf} was derived

$$L_{cf} = \mu \left\{ \frac{1+\frac{t_m}{4h}}{\frac{w+t_m}{h}} + \frac{1/\pi}{\left(\frac{w+t_m}{h}\right)^2} \left[\frac{1}{2}\left(\frac{t_m^2}{h^2} - \frac{w^2}{h^2}\right) a\tan\frac{t_m}{w} \right. \right.$$

$$+ \frac{1}{4}\left(4 - \frac{w^2}{h^2}\right) a\tan\frac{2h}{w} + \frac{1}{4}\left(\frac{w^2}{h} - 4\left(1+\frac{t_m}{h}\right)^2\right) a\tan\frac{2\left(1+\frac{t_m}{h}\right)}{\frac{w}{h}} + \left(1+\left(1+\frac{t_m}{h}\right)^2\right)\log 2$$

$$+\left(1+\frac{t_m}{h}\right)^2 \log\left(1+\frac{t_m}{h}\right) - \frac{1}{2}\left(\frac{w^2}{h^2}\log\frac{w}{h} + \frac{t_m^2}{h^2}\log\frac{t_m}{t_{ox}}\right) - \frac{w}{2h}\log\left(4+\frac{w^2}{h^2}\right) - \frac{wt_m}{2h^2}\log\frac{w^2+t_m^2}{h^2}$$

$$\left. \left. + \frac{w}{2h}\left(1+\frac{t_m}{h}\right)\log\left(\frac{w^2}{h^2} + 4\left(1+\frac{t_m}{h}\right)^2\right) + \frac{1}{4}\left(\frac{w^2}{h^2} - \left(2+\frac{t_m}{h}\right)^2\right)\log\left(\frac{w^2}{h^2} + \left(2+\frac{t_m}{h}\right)^2\right) \right] \right\} \tag{19}$$

where w is the width and t_m is the thickness of the microstrip conductor.

It should be noted that the closed-form expressions for microstrip inductance such as (19) are based on the assumption of a perfect conductors with generally nonuniform current distributions [14, 18].

3. Simulation Results

The derived closed-form model for frequency-dependent series inductance and resistance parameters of single on-chip interconnects have been applied to various cases.

In order to validate the derived new formulas, the frequency-dependent per-unit-length series inductance and resistance parameters [R(ω),L(ω)] for an microstrip interconnect on a heavily doped CMOS substrate (resistivity ρ_{si} = 0.01 Ω-cm) with a 2 μm oxide layer is considered. The width of the microstrip is 4 μm, thickness 1 μm and the thickness of the silicon substrate is 500 μm. Fig. 4 shows the variation in the distributed resistance, R(ω), as a function of frequency. Similarly, Fig. 5 shows the change in the distributed inductance, L(ω), as a function of frequency. It is observed that the values of the inductance and resistance per unit length, calculated from the new formulas, are found to be in good agreement with those of [2, 6] (full-wave quasi-TEM technique and CAD-oriented circuit modeling approach). The agreement is excellent over the entire frequency range of 0.2 - 10 GHz. The maximum deviations observed between the new analytic formulas and the EM simulation results of Figs. 4 and 5 correspond to relative error less than 3 %. As expected, the lossy silicon substrate has a significant impact on the frequency-dependent characteristics of the microstrip interconnect and must be attributed to the skin effect in the substrate (the skin effect in the conductor metal plays only a minor role and can be neglected).

4. Conclusion

As a conlusion, very accurate frequency-dependent closed-form model is proposed to calculate the distributed series impedance parameters (inductance and resistance per unit length) of single on-chip interconnects on a lossy silicon substrate over the entire frequency range (up to a few tens GHz). The results obtained by using these closed-form expressions were compared with results obtained from electromagnetic simulations as well as CAD-oriented circuit modeling approach. Their simple form enables the application in the design and validation phase of RF and microwave integrated circuits in CMOS technology.

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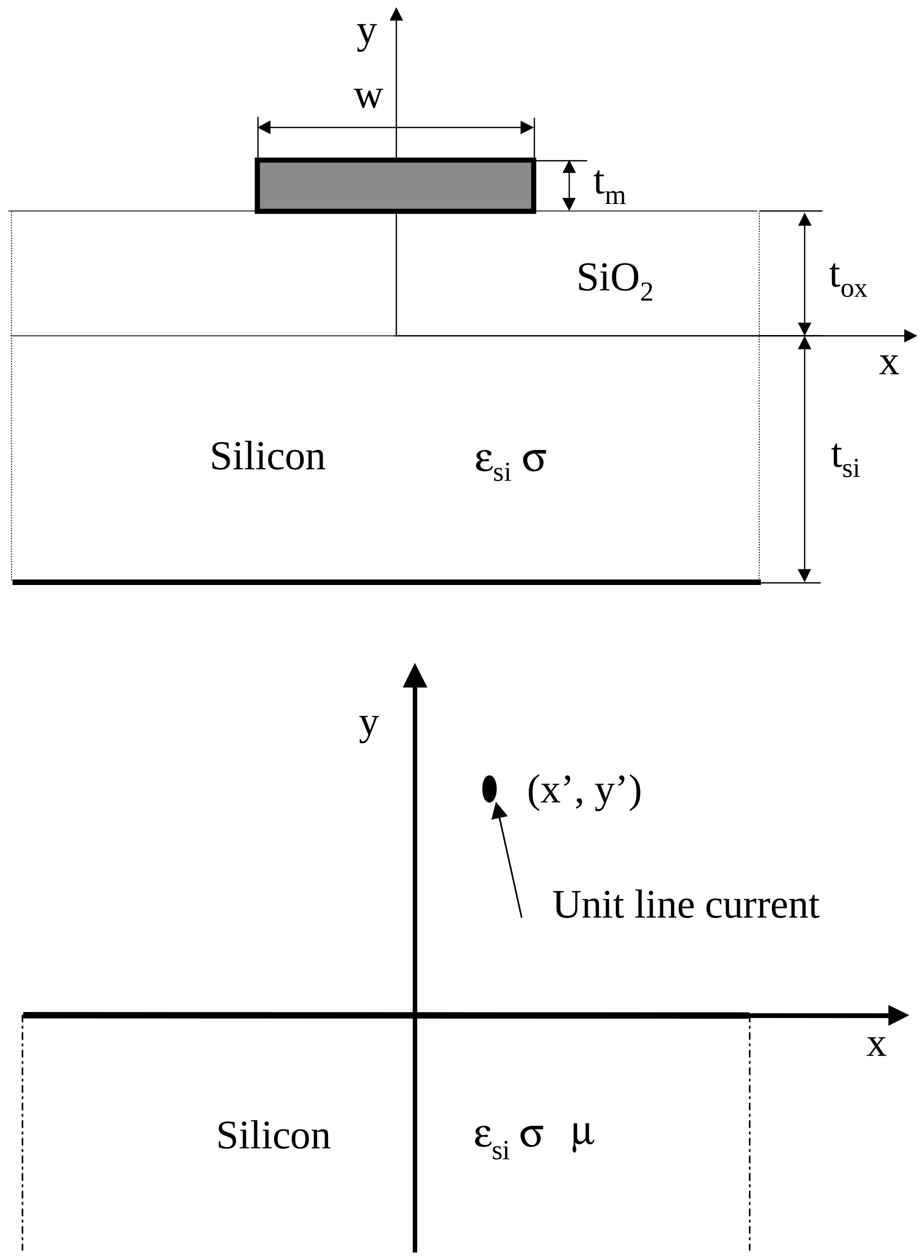

Fig. 1. (a) Single on-chip interconnect on an oxide-silicon substrate and (b) the unit current line above a semi-infinite lossy silicon substrate.

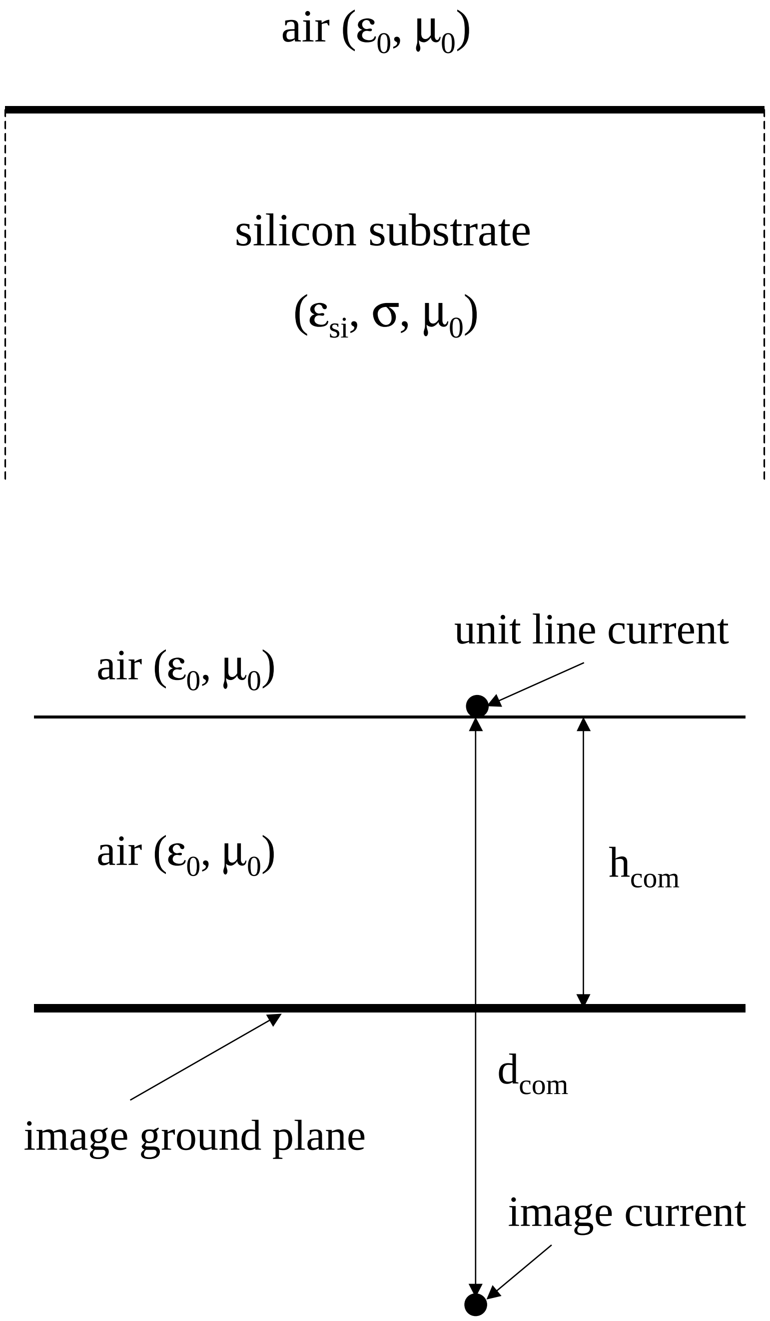

Fig. 2 Illustration of complex image procedure for structure with a lossy silicon semi-space.

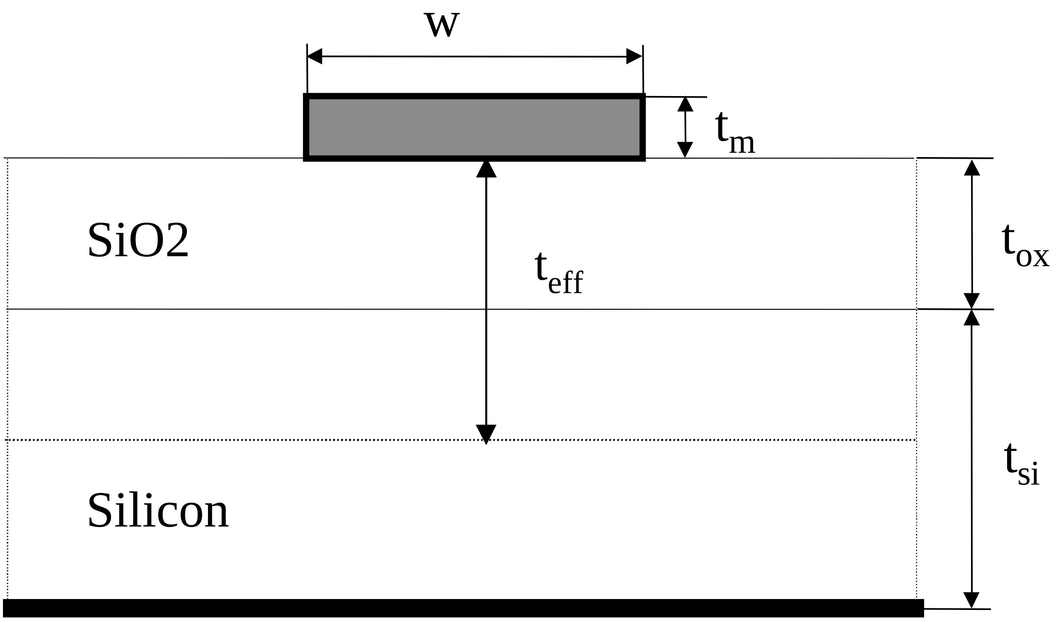

Fig. 3. Illustration of the virtual ground plane for single interconnects on lossy silicon substrate.

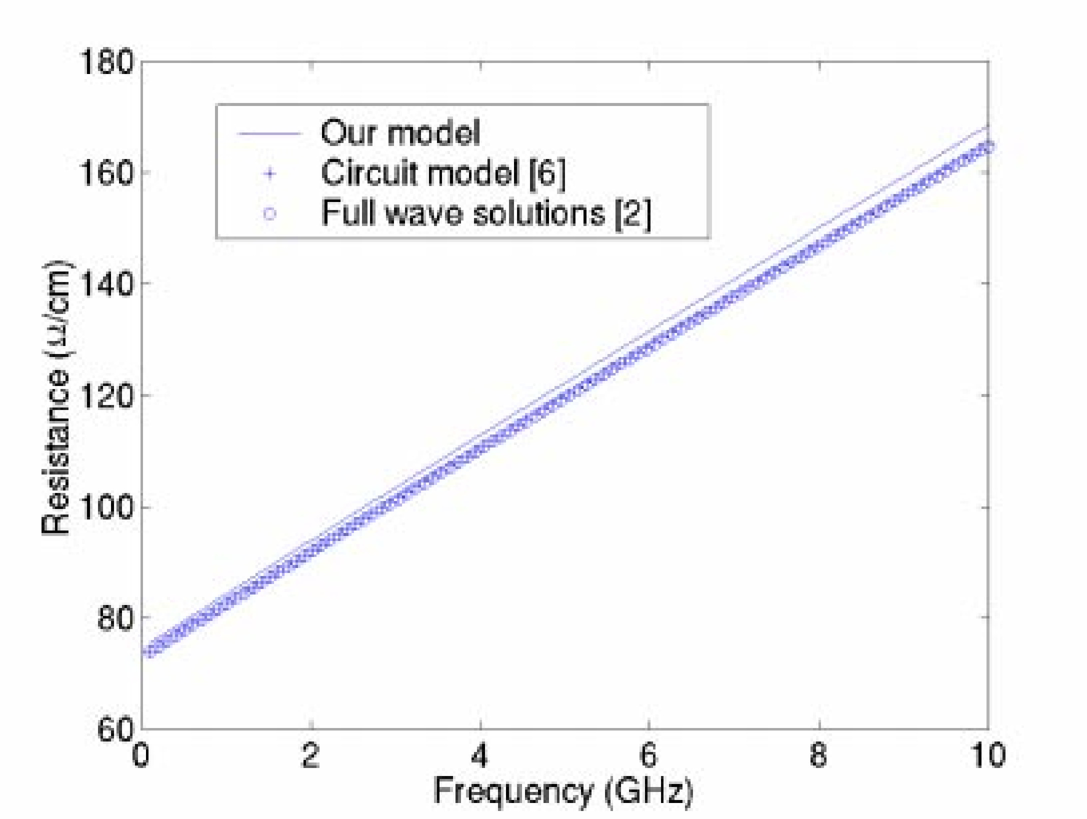

Fig. 4. Distributed series resistance per unit length of a single interconnect line on silicon substrate as the function of a frequency.

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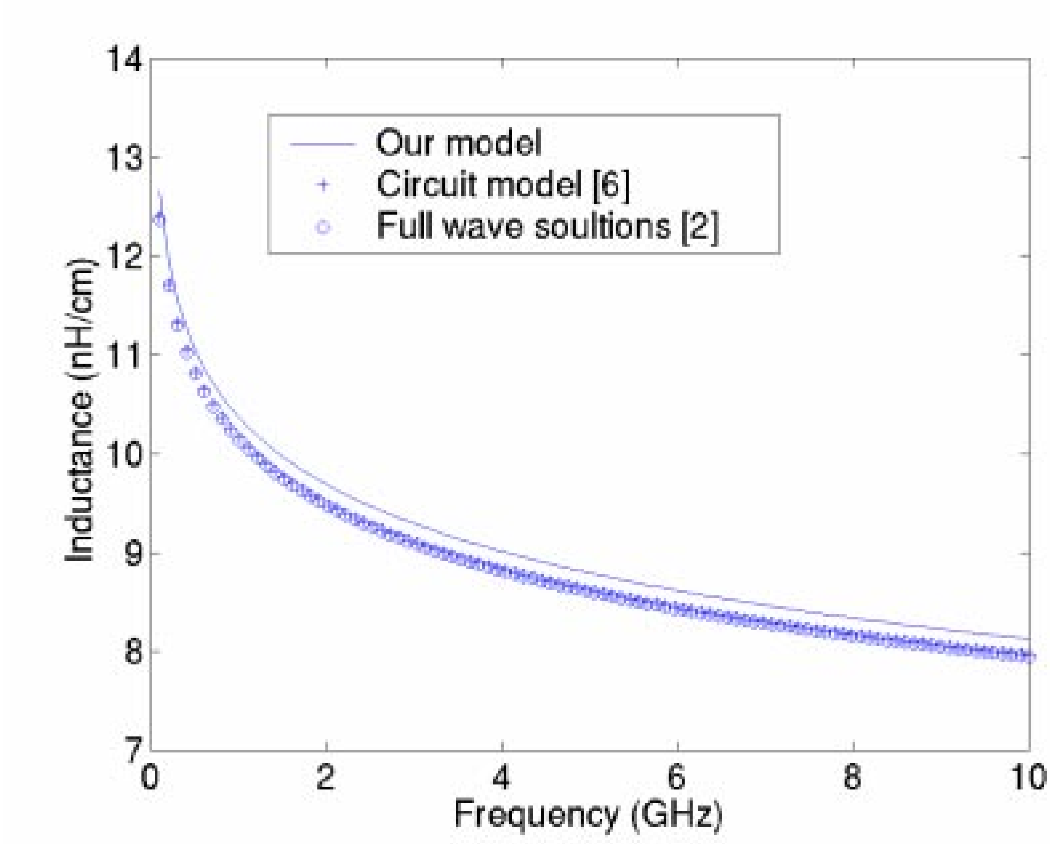

Fig. 5. Distributed series inductance per unit length of a single interconnect line on silicon substrate as the function of a frequency.