ANALYSIS OF POWER TRANSIENTS IN ERBIUM-DOPED FIBER AMPLIFIERS, WITH APPLICATION TO WAVELENGTH ROUTED OPTICAL NETWORKS

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Abstract

This paper presents a time-domain modeling of Erbium-doped fiber amplifiers (EDFAs), which is adequate for the analysis of the power transients resulting from the dynamic variation of the number of channels in wavelength routed optical networks.

I. INTRODUCTION

Optical networks based on wavelength multiplexing/routing may present a variable number of channels, either due to a dynamic reconfiguration of the network (for fault correction, for example), or to increasing the network capacity. Depending on the situation, the number of channels may go up or down.

As such optical networks rely on EDFAs, which are operated very near saturation, the output power of each channel will depend on the total number of channels being amplified. The reason for that is the fact that the output power of a saturated EDFA is practically constant, and does not depend on the number of channels [1], [2]. As the number of channels varies, the surviving channels will then experience power transients, due to gain saturation induced crosstalk in the amplifiers [3], [4]. The dynamic characteristics (speed, duration, and amplitude) of such power transients depend on the total number of EDFAs in the network.

The power transients represent a major limitation to the performance of WDM networks, whenever channels are added or dropped. If some channels are dropped, the power in the surviving channels may surpass the threshold above which the fiber nonlinearities cannot be neglected any longer. If channels are added, the power in the surviving ones diminishes, and may fall below the receiver sensitivity.

The power transients resulting from the variation of the number of channels can seriously degrade the overall performance of WDM systems and networks. The knowledge of the temporal evolution of WDM signal amplified by EDFAs is then necessary, not only for an evaluation of the

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system/network performance under varying load conditions, but also to investigate techniques to mitigate the effects of the power transients. The basic tool for that is an adequate model to describe the dynamic gain characteristics of EDFAs.

This paper presents a simplified dynamic model for EDFAs, aiming at the analysis of practical situations related to WDM systems and networks.

II. DYNAMIC MODEL FOR EDFAS

The modeling of the dynamic characteristics of an EDFA requires the solution of the rate and propagation equations at the signal and pump frequencies, and for the amplified spontaneous emission (ASE) noise [1]. In the most complete model, a system of coupled, nonlinear, partial differential equations results, which is not amenable to analytical solution. The computational cost of the numerical solution, in terms of both CPU time and memory allocation, increases in the case of long haul WDM networks with a large number of EDFAs.

Several simplified dynamic models for EDFAs can be found in the literature. For instance, reference [4] proposes an analysis based on a small signal perturbation theory. In reference [5], the analytical solution of the rate equations at the input of an EDFA is used to estimate the characteristics of the transient effects. In reference [6], the rate equations are integrated along the amplifier length, so that the system of coupled, partial differential equations is reduced to a single ordinary differential equation (ODE) for the gain of an arbitrary channel. The solution of this equation allows the calculation of the gain of all other channels. This model assumes bi-directional propagation of signal and pump fields. A similar approach is employed in reference [7], where a single ODE is obtained for the population density of the metastable level, integrated along the length of the amplifier. Once this equation is solved, the time dependent output power for all channels can be calculated. But the model in reference [7] considers only unidirectional propagation of signal and pump fields.

The dynamic model presented in this paper is based mostly on the results of reference [6], but some aspects of the model proposed in reference [7] are also used, yielding a simpler formulation, and allowing for a clearer interpretation of the physical phenomena.

The objective of this model is the description of transient effects in EDFAs, where each channel operates near saturation. In this situation, the signal intensity is much higher than that of the ASE noise, which is then neglected. The model is based on an atomic system of two levels: the basic level $^4I_{15/2}$, and the metastable level $^4I_{13/2}$, whose normalized population densities are represented as n_1 and n_2, respectively. The model considers that there is no excited state absorption (ESA), once the pump wavelength is taken at 980 nm, and it is assumed that the gain spectrum is uniformly distributed. It is worth mentioning that these conditions are satisfied whenever the amplifier gain is smaller than 20 dB, or the input powers are sufficiently high, as is the case in WDM systems. The intrinsic loss of the Erbium-doped fiber is neglected, due to its short length.

In the conditions specified above, and considering an N-channel WDM system, the rate and propagation equations are written as [6]:

$$\frac{\partial n_2(z,t)}{\partial t} = -\frac{n_2(z,t)}{\tau} - \sum_{k=p,i} \frac{u_k P_k}{h\nu_k \tau\xi} \{[\alpha_k + g_k] n_2(z,t) - \alpha_k\} \tag{1.1}$$

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$$\frac{\partial P_k(z,t)}{\partial z} = u_k\{[\alpha_k + g_k]n_2(z,t) - \alpha_k\}P_k(z,t) \tag{1.2}$$

The normalized population densities of the basic, $n_1(z,t)$, and metastable, $n_2(z,t)$, levels satisfy the following relation:

$$n_1(z,t) + n_2(z,t) = 1 \tag{1.3}$$

In the equations (1.1) and (1.2), z represents distance along the fiber, and t represents elapsed time measured in a referential frame that moves at the signal velocity; $u_k = 1$ for propagation in the positive direction (increasing z); P_k represents the pump power (k = p), as well as the power of the N channels (k = i, with i = 1,2,…,N) to be amplified; ν_k represents the frequency of field k. h is Planck constant, τ is the fluorescence lifetime in the metastable level; ξ is the saturation parameter of the fiber, given by [8]: $\xi = \pi a_d^2 n_{Er}/\tau$, where n_{Er} represents the total Erbium ion concentration, considered to be uniformly distributed in a circular region of the fiber, with radius a_d. The parameters α_k and g_k are, respectively, the absorption and emission coefficients, defined as [8]:

$$\alpha_k = \sigma_k^a \Gamma_k n_{Er} \qquad\qquad g_k = \sigma_k^e \Gamma_k n_{Er} \tag{2}$$

where the index k specifies the channel order (and frequency), the parameters σ_k^a and σ_k^e represent, respectively, the absorption and emission cross-section. The factor Γ_k, called confinement factor, represents the overlap integral of the optical field k and the dope core, being given as [1]:

$$\Gamma_k = \int_A \psi_k(x,y)\,dxdy \tag{3}$$

where $\psi_k(x,y)$ is the normalized field intensity profile at the frequency k, and the integral is performed on the fiber cross-section A. The parameters α_k and g_k are determined experimentally; the parameter ξ can be determined through direct measurement of the saturation power of the fiber, P_k^{sat} , according to $\xi = P_k^{sat}(\alpha_k + g_k)/h\nu_k$, [8]-[9]. The doped fiber is completely characterized by means of the spectral parameters α_k, g_k, and ξ [1].

$n_2(z,t)$ can be calculated in terms of $P_k(z,t)$ in the equation (1.2), and substituted back in equation (1.1). A propagation equation in terms of $P_k(z,t)$ is then obtained:

$$\tau u_k \frac{\partial}{\partial t}\left(\frac{1}{P_k(z,t)}\frac{\partial P_k(z,t)}{\partial z}\right) = -\alpha_k - u_k\frac{1}{P_k(z,t)}\frac{\partial P_k(z,t)}{\partial z} - \frac{1}{P_k^{IS}}\sum_{j=p,i}\frac{u_j}{\nu_j}\frac{\partial P_k(z,t)}{\partial z} \tag{4}$$

where $P_k^{IS} = h\xi/(\alpha_k + g_k)$ is the intrinsic saturation power of the k-th channel [8].

Integration of the equation (4) in z, from z=0 to z=L (length of the doped fiber) yields:

$$\tau u_k \frac{d}{dt}\left[\ln\left(\frac{P_k^L(t)}{P_k^0(t)}\right)\right] = -u_k \ln\left(\frac{P_k^L(t)}{P_k^0(t)}\right) - \frac{1}{P_k^{IS}} \sum_{j=p,i} \frac{u_j}{\nu_j}\left[P_j^L(t) - P_j^0(t)\right] - \alpha_k \qquad (5)$$

with $P_k^0(t) = P_k(z = 0, t)$ e $P_k^L(t) = P_k(z = L, t)$.

Defining the total gain parameter $G_k(t) = \ln\left[P_k^L(t)/P_k^0(t)\right]$ and the total absorption constant $A_k = \alpha_k L$, equation (5) is re-written as:

$$P_k^{IS}\left[\tau u_k \frac{dG_k(t)}{dt} + u_k G_k(t) + A_k\right] = -\sum_{j=p,i} \frac{u_j P_j^0(t)}{\nu_j}\left\{\exp\left[G_j(t)\right] - 1\right\} \qquad (6)$$

This equation represents a set of N+1 coupled ODEs, and describes the temporal evolution of pump and N channel fields.

Knowing the initial powers $P_k^0(t)$ and the initial conditions $G_k(t = 0) = G_k^0$ for all fields, pump (k = p) and signal channels (k = 1,2,…,N), the system of equations (6) can be solved numerically, allowing for the calculation of the temporal evolution of the total gain $G_k(t)$, for all values of k, and so the study of transient effects in EDFAs. However, this system of coupled ODEs can be simplified to a single equation noting that the r.h.s. of equation (6) is the same for all values of k. Combination of any two equations, say for fields k and j, the following first order ODE is obtained:

$$\tau u_k \frac{dG_{kj}(t)}{dt} + u_k G_{kj}(t) + A_{kj} = 0 \qquad (7)$$

where $G_{kj}(t) = P_k^{IS} G_k(t) - P_j^{IS} G_j(t)$ and $A_{kj} = P_k^{IS} A_k - P_j^{IS} A_j$.

The solution of equation (7) is easily calculated as [6]:

$$G_{kj}(t) = \left(G_{kj}^0 + A_{kj}\right)\exp\left(\frac{-t}{\tau}\right) - A_{kj} \qquad (8)$$

where the initial conditions are used to determine $G_{kj}^0 = P_k^{IS} G_k^0 - P_j^{IS} G_j^0$, $G_j^0 = G_j(t = 0)$.

With the solution of equation (8), the gain of channel j can be expressed in terms of the gain of channel k:

$$P_j^{IS} G_j(t) = P_k^{IS} G_k(t) - \left(G_{kj}^0 + A_{kj}\right)\exp\left(\frac{-t}{\tau}\right) + A_{kj} \qquad (9)$$

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Substitution of the equation (9) in the equation (6) yields a single ODE for the gain of channel k:

$$\tau\frac{d}{dt}\left[P_k^{IS}G_k(t)\right]+P_k^{IS}G_k(t)+P_k^{IS}A_k=-\sum_{j=p,i}\frac{P_j^0}{\nu_j}\left\{\exp\left[\left(P_k^{IS}G_k(t)-(G_{kj}^0+A_{kj})\exp(\frac{-t}{\tau})+A_{kj}\right)/P_j^{IS}\right]-1\right\} \quad (10)$$

Once the gain for a given channel k has been calculated from the equation (10), the gain for all other channels can be obtained via the equation (9).

The solution of the equation (10) requires the initial values of the pump and signal powers at the input of the Erbium-doped fiber, P_j^0, as well as the values of the gain G_j^0 (j = p for the pump, and j=1,2,…,N for the signal channels). The specification of these initial conditions defines, as a matter of fact, the type of analysis to be carried out, as explained in the next section.

III. NUMERICAL SIMULATION

A computer program was developed for the solution of the equation (10), and to determine the temporal variation of the output power and gain for the signal channels in a WDM system.

The computer program uses three sets of input data: the first set of data specifies the physical characteristics of the Erbium-doped fiber – length L (in meters), fiber saturation parameter ξ ($m^{-1}s^{-1}$), and fluorescence lifetime in the metastable level τ (s).

The second set of data specifies the parameters for the numerical simulation: width of the time window (s), time step (s), modulation index amplitude (to simulate the dropping of a channel), number of EDFAs along the link, number of channels in the system, order of the channels to be added/dropped, and the type of initial conditions.

The third set of input data carries the values of the power (W), wavelength (m), gain g_k (m^{-1}) and absorption α_k (m^{-1}) coefficients for the signal channels and pump, as well as the pumping scheme, which can be either in the same (u_k=1), or in the opposite (u_k=-1) direction of the signal propagation.

Two types of analysis are possible, and the choice between the two specifies the initial conditions to be used for the solution of the equation (10). The first type of analysis considers the temporal evolution of the signal channel gain from the instant the amplifier is excited by the pump and signals. In this case, the initial conditions are taken as $P_j^0=G_j^0=0$.

The second type of analysis, of great practical interest, considers that pumping has already been applied to the amplifier prior to the analysis, together with a given number of signals in different wavelengths. In this case, the term $\left(G_{kj}^0+A_{kj}\right)=0$ must be substituted in the equations (9) and (10). This situation is maintained for a certain period of time, until a (new) signal is (added) dropped, and, after a while, (dropped) added back to the system. This type of analysis allows for the study of transient effects in EDFAs, which can result from dropping or adding channels. The initial conditions for the solution of the equation (10) are obtained from the results

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of reference [7]. It is assumed that before a new signal is added (or one of the original signals is dropped), the amplifier is in stationary conditions.

The values of G_j^0 are obtained as [7]:

$$G_j^0 = (\alpha_j + g_j)L\rho - \alpha_j L \quad , \quad j=1,\ 2, ..., N \tag{11.1}$$

$$G_p^0 = -\alpha_p L(1-\rho) \tag{11.2}$$

where the parameter $\rho = L^{-1}\int_0^L n_2(z, t=0)dz$ is obtained from the solution of the following equation [7]:

$$\sum_{j=1}^{N} p_j^0 \left\{ \frac{\exp\left[(g_j + \alpha_j)L\rho - \alpha_j L\right] - 1}{L} \right\} + \rho - u_p p_p^0 \left\{ \frac{1 - \exp\left[-\alpha_p L(1-\rho)\right]}{L} \right\} = 0 \tag{12}$$

with $p_j^0 = \dfrac{P_j^0}{hv_j \xi} \quad (j = p, 1, 2, ..., N)$.

Once the initial conditions have been established, the equation (10) is solved numerically via a forth-order Runge-Kutta algorithm [10].

In the case of a cascade of EDFAs, the initial conditions must be re-established at the input of each amplifier, which compensates exactly for the loss of the fiber segment that precedes it. The initial condition for the pump is the same in all amplifiers, but the values of the signal powers at the input of a given amplifier depend on the values of the output powers at the previous amplifier.

In the examples to follow, the following set of parameters were used:

Length of the Erbium-doped fiber: 12.0 m
Fiber saturation parameter: $1.099\text{x}10^{15}$ $m^{-1}s^{-1}$ Parameter τ_{21}: $11.0\text{x}10^{-3}$ s
Parameter α_{13}: $8.24\text{x}10^{-0.01}$
Parameter g_{21}: 0
Pump wavelength: $0.982\text{x}10^{-6}$ m
Pump power: $40.0\text{x}10^{-3}$ W

As a matter of convenience, eight channels were chosen with wavelengths between 1539 nm and 1553 nm, equally spaced by 2.0 nm, and denoted channel no. 1, 2, …, 8, respectively. In the examples to follow, one or more of these channels, the choice being totally arbitrary, were propagated along the link. In all the examples, co-propagating pumping was considered, with the pump power given above.

Initially, the case of the excitation of an EDFA by channels no. 1, 2, 3, and 4 (wavelengths 1539 nm, 1541 nm, 1543 nm, and 1545 nm, respectively), with –6.0dBm power in each channel,

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is considered. Figure 1 illustrates, at the output of the amplifier, the temporal evolution of the signal powers, from the instant of excitation of the amplifier. It is clear that the amplifier requires a certain period of time to reach stationary conditions, about 40 ms. The gain obtained for each channel is 13dB. This type of analysis provides useful information for the design of optical amplifiers. For example, one can investigate how the final gain depends on physical and/or geometrical parameters of the Erbium-doped fiber.

The next examples illustrate transient effects in EDFAs, which result from the adding or dropping of channels. In these examples, it is assumed that the amplifier has reached stationary conditions before a channel is added/dropped.

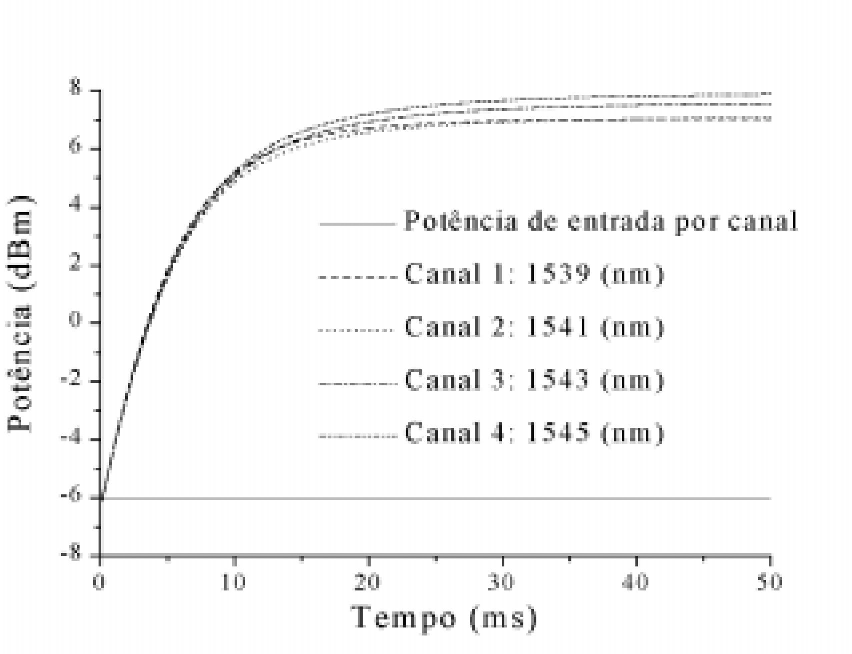

Figure 1: Temporal evolution of the output power of the channels no. 1, 2, 3, and 4, considering a single EDFA.

Figure 2 corresponds to the case of the propagation of the channels no.1 and no.2, each one with an input power of 250 μW. The channel no. 2 is dropped, and latter added back. The effect of gain saturation induced crosstalk is clearly seen in the figure. Immediately after the dropping of the channel no. 2, the output power of the channel no. 1 (i.e. the amplifier gain) increases rapidly, until it reaches a stable value. When the channel no. 2 is added back, the gain of the amplifier for both channels is altered again, until the initial values (before the channel no. 2 was dropped) are recovered. To help visualization, the values of the input power of the channel no. 2 were increased 30 times.

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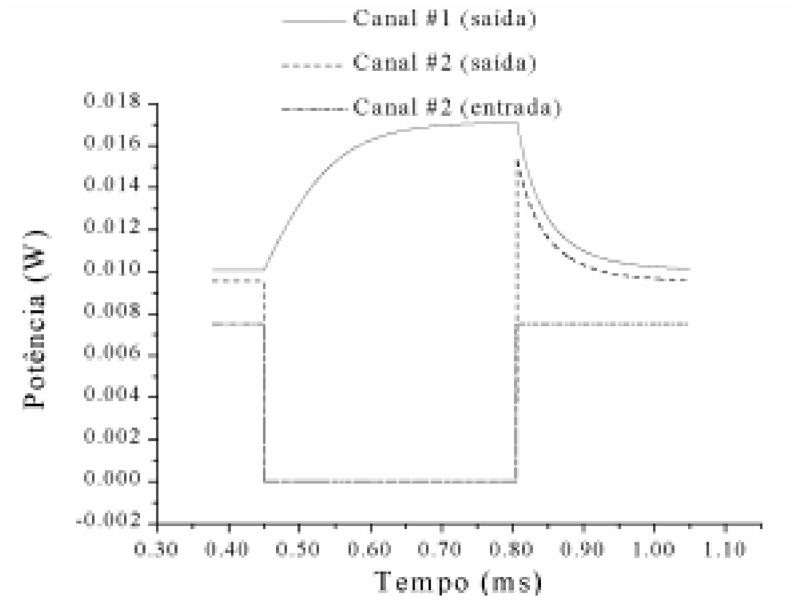

Figure 2: Output power of channels no. 1 (1539nm) and no. 2 (1541nm), when the channel no. 2 is dropped, and later added back to the system.

The previous examples considered only the propagation of signals through a single EDFA. The characteristics of the transient effects change considerably when the signals propagate through a cascade of optical amplifiers: the time scales associated to the power transients are reduced in the inverse proportion of the number of amplifiers.

The next examples illustrate this effect for a cascade of 12 EDFAs. In these examples, the fiber segment between successive amplifiers is treated as a simple attenuator, the loss of which is compensated exactly by the gain of the following amplifier.

Initially, 4 channels (channels no. 1, no.2, no.3, no.4) are propagated through the cascade of 12 EDFAs. The initial power of each channel is -6 dBm. Once the amplifiers reach stationary operating conditions, the channels no. 2 and no. 4 are dropped. Figure 3 shows the temporal evolution of the four channels at the output of the 12th amplifier. It is clear that the output power of the surviving channels is affected. The transient period until a new stationary condition is reached is clearly seen in Figure 3. It is also apparent that immediately after the dropping of the channels no.1 and no. 2, the power of the surviving channels increases suddenly. This power variation will certainly influence the overall performance of the system.

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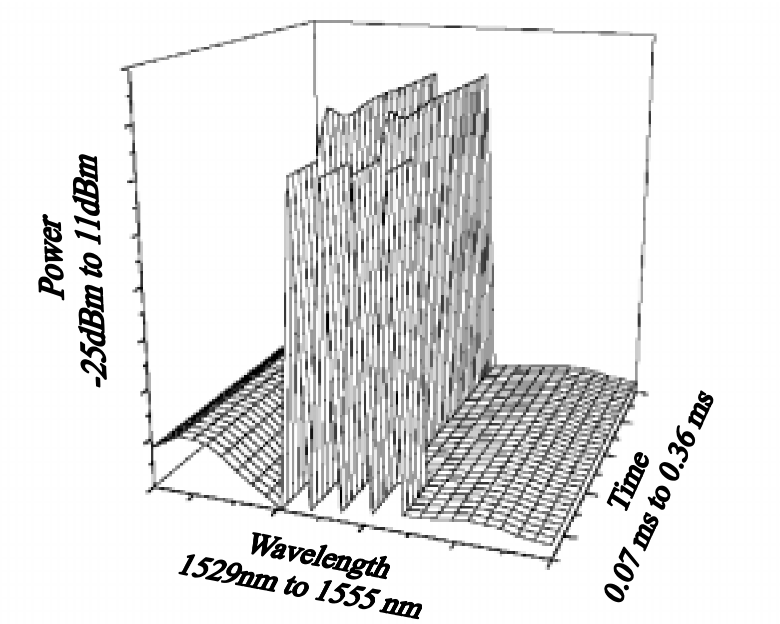

Figure 3: Temporal evolution of the signals at the output of the 12th EDFA, after dropping channels no. 2 and 4.

The next example considers the propagation of all the eight channels through the cascade of 12 EDFAs. Figure 4 illustrate the variation of the total output power at the output of the even-order amplifiers, when four channels are dropped after stationary operating conditions were reached. The channels no. 2, 4, 6, and 8 are dropped, while the channels no. 1, 3, 5, and 7 remain in the system.

Figure 4 shows that the transient effects become more rapid along the cascade of EDFAs. For example, at the output of the 10th EDFA, the peek transient occurs within 0.03 ms, while at the output of the 2nd EDFA this peek occurs within 0.13 ms. This figure also shows that the intensity of the transient peek increases along the cascade of amplifiers.

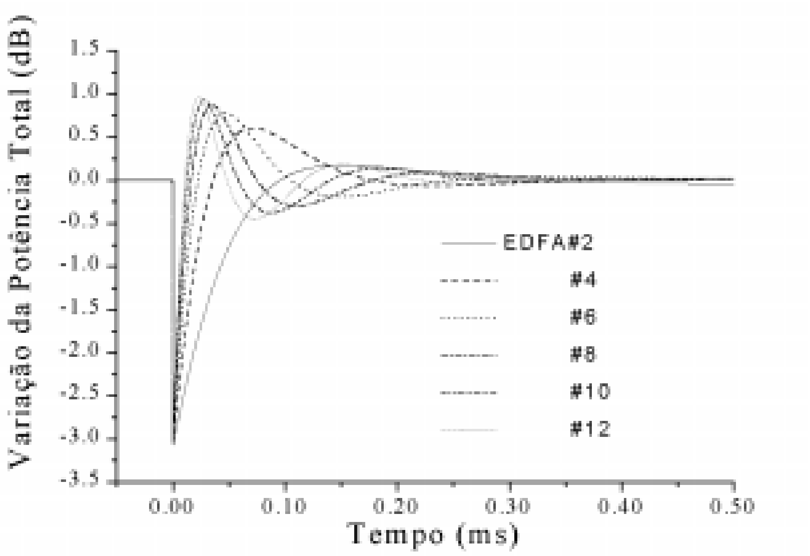

Figure 4: Variation of the total power at the output of the even-order amplifiers, when channels no. 2, 4, 6, and 8 are dropped at t=0.

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Figure 5 shows the case were the channels no. 2, 4, 6, and 8 are added to the system, considering that the channels no. 1, 3, 5, and 7 were already present and had reached stationary conditions. The curves in this figure indicate that the transients that result from the adding of channels have the same time scale as those resulting from the dropping of channels. But when channels are added to the system, there is a reduction of the total output power.

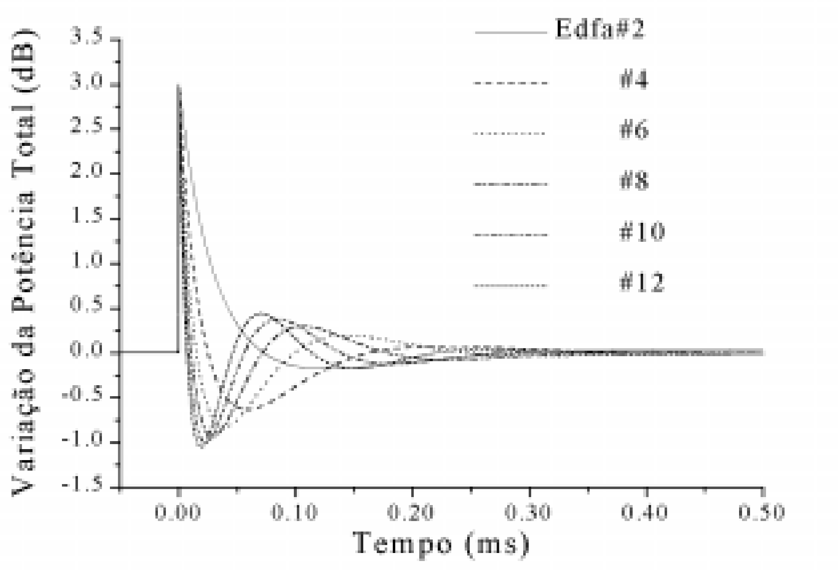

Figure 5: Variation of the total power at the output of the even-order amplifiers, when the channels no. 2, 4, 6, and 8 are added at t=0 (the channels no. 1, 3, 5, and 7 were already present at t<0, and were kept in the system).

IV. CONCLUSION

A numerical model was developed for the dynamic analysis of Erbium-doped fiber amplifiers. The model was applied to the analysis of transient effects in a single EDFA, as well as in a cascade of 12 EDFAs. The results show clearly that the adding or dropping of channels in a WDM system gives origin to power transients in the surviving channels. The duration of such transients decreases as the number of amplifiers in the cascade increases. The maximum intensity of these transients increases with the number of amplifiers. Depending on the initial power of each channel, and on the number of amplifiers in the cascade, the power transient of the surviving channels can severely affect the performance of WDM communication systems.

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