Polarimetric Bistatic Scattering
From Random Rough Surfaces
Along Azimuth Angle

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Abstract

Electromagnetic wave interaction with random media is a process which needs improved theoretical representations. Existing assumptions and approximations for electromagnetic bistatic scattering from randomly rough surfaces sometimes remain unjustified, since this scattering phenomena is not fully understood. For further understanding the scattering behavior of electromagnetic waves from randomly rough surfaces we need an improved model to predict the bistatic scattering behaviors along different azimuth angle. The model developed for predicting the scattering characteristics from statistically rough surfaces in this paper is based upon the integral equation model with multiple scattering. Its objective is to examine the bistatic scattering characteristics from randomly rough surfaces along the different azimuth angle. For better understanding the scattering behavior from randomly rough surfaces, the information about scattering must be known omnidirectionally, not just in the specular or backscatter direction. Comparisons between the model prediction and the experimental measurements are made for both like and cross polarizations. This model prediction demonstrates good agreement with the experimental data.

Introduction

Electromagnetic wave scattering from randomly rough surfaces is applicable to research interest in many areas including remote sensing, ocean acoustics, imaging of biological optics and surface optics. Earlier work on surface scattering mainly focused on the scattering from one-dimensional random rough surface and fruitful results were reported in various numerical, analytical and experimental studies. However the roughness of most naturally occurring surfaces is two-dimensional instead of one-dimensional in nature. In order to address practical application-oriented situations, the research on scattering from two-dimensional surfaces has become important. Because of the incoherent complexities involved, a limited model predicts the bistatic scattering behaviors, especially out of plane of incidence. More recently interest has arisen in bistatic surface scattering because it is possible to measure the signal from the global positioning system after it is scattered out of plane of incidence from the ocean surface [16-18]. For this reason, in this paper we developed a model to study the bistatic scattering trend from two-dimensional random rough surfaces out of plane of incidence. The model developed based upon the integral equation method and corrected by the angular and propagation shadowing functions in this paper. The shadowing functions are modified and applied to the single and multiple scattering.
To account the bistatic scattering properties out of plane of incidence we develop a formulation for estimating the tangential surface fields on the randomly rough dielectric surfaces. Then, the scatter fields can be computed in terms of the tangential surface fields. Subsequently, the average scattered power and the scattering coefficients can be obtained. After developing the surface scattering model along any azimuth angle, we evaluate these integrals and obtained practically useful surface scattering models under various conditions [16]. For convenience in mathematical evaluation we provide two different forms for the surface scattering model depending upon whether the surface height is moderate or large in terms of the incident wavelength. Based on the integral equation method the bistatic scattering coefficients for like and cross polarizations were shown in section 2. The scattering predictions in the plane of incidence and orthogonal to the plane of incidence are shown in section 3. The scattering behaviors along the different azimuth angle are also studied. Finally the comparisons of model predictions with the measured data are made.

Model development for polarized bistatic scattering

The governing equations for the tangential surface fields on a dielectric surface have been given by Poggio and Miller [6]. The tangential surface electrical and magnetic fields in the upper are

\[
\hat{n} \times \vec{E} = 2\hat{n} \times \vec{E}^i - \frac{2}{4\pi} \hat{n} \times \int \vec{E} ds'
\]

and

\[
\hat{n} \times \vec{H} = 2\hat{n} \times \vec{H}^i + \frac{2}{4\pi} \hat{n} \times \int \vec{H} ds'
\]

and in the lower medium are

\[
\hat{n}_r \times \vec{E}_r = -\frac{2}{4\pi} \hat{n}_r \times \int \vec{E}_r ds'
\]

\[
\hat{n} \times \vec{H}_r = \frac{2}{4\pi} \hat{n}_r \times \int \vec{H}_r ds'
\]

where

\[
\vec{E} = jk\eta (\hat{n}' \times \vec{H}') G - (\hat{n}' \times \vec{E}') \nabla' G - (\hat{n}' \cdot \vec{E}') \nabla' G
\]

\[
\vec{H} = \frac{jk}{\eta} (\hat{n}' \times \vec{E}') G - (\hat{n}' \times \vec{H}') \nabla' G + (\hat{n}' \cdot \vec{H}') \nabla' G
\]

The fields in the lower medium can be written in terms of the fields in the upper medium by applying the boundary conditions on the continuity of the tangential fields. The spectral representation for the Green’s function and its gradient is
\[ G = \left( -\frac{1}{2\pi} \right) \int \frac{j}{q} \exp[ju(x-x')+jv(y-y')-jqz-z'] dudv \] (7)

and

\[ \nabla'G = \left( -\frac{1}{2\pi} \right) \int \frac{\hat{g}}{q} \exp[ju(x-x')+jv(y-y')-jqz-z'] dudv \] (8)

where \( q = \sqrt{k^2 - u^2 - v^2} \) and \( \hat{g} = \hat{x}u + \hat{y}v \pm \hat{z}q \). \( z \) and \( z' \) are the random variables representing the surface height at different locations on surface. Without the absolute value term in the Green’s function the ensemble average is the standard characteristic function for two, three and four random variables.

To find the integral equations suitable for obtaining an estimate of the tangential fields, we add and subtract \( n \times E' \) on the right-hand side of the tangential electric field equation defined by (1). Similarly, we add and subtract \( n \times H' \) on the right-hand side of the tangential magnetic field equation given by (2), where \( E' \) and \( H' \) are the reflected electric and magnetic fields propagating along the reflected direction, \( \hat{k}' \). Thus, a pair of surface integral equations is obtained as follows

\[ \hat{n} \times \hat{E} = (\hat{n} \times \hat{E})_k + (\hat{n} \times \hat{E})_c \] (9)

\[ \hat{n} \times \hat{H} = (\hat{n} \times \hat{H})_k + (\hat{n} \times \hat{H})_c \] (10)

where \( (\hat{n} \times \hat{E})_k \) and \( (\hat{n} \times \hat{H})_k \) are the tangential fields under the Kirchhoff approximation, i.e.,

\[ (\hat{n} \times \hat{E})_k = \hat{n} \times (\hat{E} + \hat{E}') \] (11)

\[ (\hat{n} \times \hat{H})_k = \hat{n} \times (\hat{H} + \hat{H}') \] (12)

and \( (\hat{n} \times \hat{E})_c \) and \( (\hat{n} \times \hat{H})_c \) are defined as

\[ (\hat{n} \times \hat{E})_c = \hat{n} \times (\hat{E} - \hat{E}') - \frac{2}{4\pi} \hat{n}_i \times \hat{E} ds' \] (13)

\[ (\hat{n} \times \hat{H})_c = \hat{n} \times (\hat{H} - \hat{H}') + \frac{2}{4\pi} \hat{n}_i \times \hat{H} ds' \] (14)

The first term on the right-hand side of (13) is identical to the tangential Kirchhoff field. It requires a complementary term, which is the second term on the right-hand side of (13) to form the total tangential surface field. Thus, it is clear that the Kirchhoff tangential field alone cannot provide a good estimate to the total tangential surface field. Similar remarks are applicable to (14). To use (13) and (14) for estimating the total tangential field we need to express both the Kirchhoff field and the complementary field in terms of the incident field components and the surface reflectivity properties.

With the given Kirchhoff and complementary scattered field, the ensemble average scattered
power is given by

\[ E_{qp}^s E_{qp}^* = E_{qp}^k E_{qp}^k + 2 \text{Re}\{E_{qp}^c E_{qp}^c\} \tag{15} \]

where Re means the real part operator and * is the symbol for complex conjugate. To obtain the incoherent power, we subtract the mean-squared power from the total power. That is,

\[ E_{qp}^s E_{qp}^* - E_{qp}^s E_{qp}^* = E_{qp}^k E_{qp}^k - E_{qp}^k E_{qp}^k + E_{qp}^c E_{qp}^c - E_{qp}^c E_{qp}^c + 2 \text{Re}\{E_{qp}^c E_{qp}^c\} \tag{16} \]

The incoherent scattered power includes the Kirchhoff, cross and complementary scattered power. To carry out the ensemble average operation we assume Gaussian height distribution here. The incident and scattered wave numbers are defined as \( \hat{k}_s = \hat{x}k_x + \hat{y}k_y + \hat{z}k_z \) and \( \hat{k}_i = \hat{x}k_x + \hat{y}k_y - \hat{z}k_z \). The ensemble average scattered Kirchhoff, Cross and complementary terms are represented below respectively. The Kirchhoff ensemble average scattered power is

\[ P_{qp}^k = \left| CE_{f_{qp}} \right|^2 \left\{ \langle \int \exp\left[j(k_s - k_i) \cdot (r - r')\right] dx' dy' dx dy > - \langle \int \exp\left[j((k_i - k_s) \cdot r')\right] dx' dy' dxdy > \right\} \tag{17} \]

The cross scattered power is

\[ P_{qp}^{kc} = 2 \text{Re}\{E_{qp}^c E_{qp}^c\} = \left| CE_{f_{qp}} \right|^2 \text{Re}\{\int (F_{qp} f_{qp})\} \int \left\{ \langle \int \exp[j(k_i \cdot r - k_s' \cdot r) + jk'_s \cdot (r'' - r) + ju(x - x') + jv(y - y') - jq(z - z')] > \right\} \tag{18} \]

and the complementary scattered power is
After the lengthy calculation of ensemble average, the average scattered power of Kirchhoff, cross and complementary terms can be found sequentially. The incoherent Kirchhoff scattered power is

$$P_{\text{ap}}^k = \left\langle E_{\text{ap}}^* E_{\text{ap}} \right\rangle - \left\langle E_{\text{ap}}^* \right\rangle \left\langle E_{\text{ap}} \right\rangle$$

$$= \left| CE_o \right|/(8\pi^2) \operatorname{Re}\left\{ \int F_{\text{ap}}^* F_{\text{ap}} \right\}$$

$$\int \int \int \int \exp[jk \cdot (\vec{r} - \vec{r}')] + jk' \cdot (\vec{r}' - \vec{r})' + ju(x - x') - ju'(x'' - x''')$$

$$+ jv(y - y') - jv'(y'' - y''') - jqk - z| + jq'z'' - z'''| >$$

$$dxdy'dx''dy''dudv'dv'$$

$$- \left| \int \int \int F_{\text{ap}} \exp[jk \cdot r - jk' \cdot r' + ju(x - x') + jv(y - y') - jqk - z|]$$

$$dxdy'du'dv'>$$

where $\sigma^2$ is the variance of the surface and $\rho(x - x', y - y')$ is the normalized surface autocorrelation function. For simplicity we refer to $\rho$ as the surface correlation. Further we assume the surface is generated by a stationary random process and let $\xi = x - x'$, $\zeta = y - y'$. $A_o$ is the illuminated area.

The incoherently scattered power for the cross term is

$$P_{\text{ap}}^{k_c} = \frac{1}{2} \left| CE_o \right|/(2\pi)^2 \operatorname{Re}\left\{ \int F_{\text{ap}}^* F_{\text{ap}} \right\}$$

$$\int \{ \exp[\sigma^2 (k_{x_c} + k_{x_c'})] \rho(\xi, \zeta) - 1 \} \exp[j(k_{x_c} - k_{x_c'})\xi + j(k_{y_c} - k_{y_c'})\zeta]d\xi d\zeta$$

$$\text{where}

\begin{align*}
    k(q) &= \exp[-\sigma^2 (k_{x_c}^2 + k_{x_c'}^2 + k_{y_c} + q^2 - qk_{x_c} + qk_{x_c'})] \\
    k_{x_c}(q) &= \exp[-\sigma^2 (k_{x_c}k_{x_c'} - q^2 + qk_{x_c'} + qk_{x_c})] \rho(\xi', \zeta - \zeta') \\
    k_{x_c'}(q) &= \exp[\sigma^2 (k_{x_c} - q)(k_{x_c'} + k_{x_c'})] \rho(\xi, \zeta') \\
    k_{y_c}(q) &= \exp[\sigma^2 (k_{y_c} + q)(k_{y_c'} + k_{y_c'})] \rho(\xi, \zeta')
\end{align*}$$

and $\xi = x - x''$, $\zeta = y - y''$, $\xi' = x' - x'''$, $\zeta' = y' - y'''$. Finally the incoherently scattered power for the complementary term is
\[ P_{ap} = \frac{1}{4} |CE_e|/8 \pi^2 A_0 \int \int F_{ap} F_{ap}^* \{ f(q,q') f_1(q) f_3(q') \] 
\[ \{ f_2(q,q') f_2(q,q') \} f_1(q,q') \{ f_3(q,q') \} - 1 \} + f(q,-q') f_1(q) f_3(q') \] 
\[ \{ f_2(q,q') f_2(q,q') \} f_1(q,q') \{ f_3(q,q') \} - 1 \} + f(-q,q') f_1(q) f_3(q') \] 
\[ \{ f_2(-q,q') f_2(-q,q') \} f_1(-q,q') \{ f_3(-q,q') \} - 1 \} + f(-q,-q') f_1(-q) f_3(-q') \] 
\[ \{ f_2(-q,q') f_2(-q,q') \} f_1(-q,-q') f_3(-q,-q') - 1 \} ] \cdot e^{i(k_{xx} + k_{xy}) \xi + jk_{xy} \tau + (v-v') \kappa} \} \] 
\[ d\xi d\zeta d\xi' d\zeta' d\xi d\zeta \] 

where \( \xi = x - x'' \), \( \zeta = y - y'' \), \( \xi' = x' - x'' \), \( \zeta' = y' - y'' \), \( \tau = x'' - x''' \), and \( \kappa = y'' - y''' \)

**Bistatic scattering coefficients**

The bistatic scattering coefficient is related to the ensemble average scattered power expression as

\[ \sigma_{ap}^0 = (4\pi R^2 P_{ap}) / (E_0^2 A_0) \]  

(24)

The incoherent ensemble average scattered power can be expressed as the summation of Kirchhoff, cross and complementary scattered power. Therefore the bistatic scattering coefficient can be summarized by Kirchhoff, cross and complementary scattered coefficient.

\[ \sigma_{ap}^0 = \sigma_{ap}^k + \sigma_{ap}^c + \sigma_{ap}^c \]  

(25)

where the Kirchhoff term is

\[ \sigma_{ap}^k(s) = \frac{k^2}{4\pi} \int |f_{ap}|^2 e^{-\sigma^2(k_{xx} + k_{xy})^2} \int [e^{\sigma^2(k_{xx} + k_{xy})^2 \rho(\xi,\zeta)} - 1] \cdot e^{i(k_{xx} - k_{xy})\xi + jk_{xy} + k_{xy})\xi} \] 

\[ d\xi d\zeta \]  

(26)

the cross term is

\[ \sigma_{ap}^c(s) = \frac{k^2}{32\pi^3} \text{Re} \left[ \int \{ f_{ap} \}^* F_{ap}(-k_{xx},-k_{xy}) \right] \cdot \int \{ k(q)k_1(q)[k_2(q)k_3(q) - 1] + k(q)k_1(q)[k_2(q)k_3(q) - 1] \} \] 

\[ e^{i(k_{xx} + k_{xy})\xi + jk_{xx} + jk_{xy})\xi} \] 

\[ \frac{d\xi d\zeta}{d\xi d\zeta} \] 

where

\[ k(q) = \exp[-\sigma^2(k_{xx}^2 + k_{xy}^2 + k_{xx} + q^2 - qk_{xx} + qk_{xy})] \] 

\[ k_1(q) = \exp[-\sigma^2(k_{xx} - q^2 + qk_{xx} - qk_{xy})] \] 

\[ k_2(q) = \exp[\sigma^2(k_{xx} - q)[k_{xx} + k_{xy})] \] 

\[ k_3(q) = \exp[\sigma^2(k_{xx} + q)[k_{xx} + k_{xy})] \]
and the complementary term is

\[
\sigma_{qp} = \frac{1}{4} k^2/(16\pi^2) \int \int F_{qp} F_{qp} \left\{ f(q, q') f_1(q) f_0(q') \right\}
\]

\[
\{ f_2(q, q') f_1(q, q') f_0(q, q') f_5(q, q') - 1 \} + f(q, -q') f_1(q) f_0(-q')
\]

\[
\{ f_2(q, -q') f_1(q, -q') f_0(q, -q') f_5(q, -q') - 1 \} + f(-q, q') f_1(-q) f_0(q')
\]

\[
\{ f_2(-q, q') f_1(-q, -q') f_0(-q, q') f_5(-q, q') - 1 \} + f(-q, -q') f_1(-q) f_0(-q')
\]

\[
\{ f_2(-q, -q') f_1(-q, -q') f_0(-q, -q') f_5(-q, -q') - 1 \}
\]

\[
\exp \{ J(k_{sx} + u) \xi + (k_{sy} + v) \zeta - (k_x + u) \xi' - (k_y + v) \zeta' + (u - u') \tau + (v - v') \kappa \}
\]

\[
d\xi d\zeta d\xi' d\zeta' d\tau d\kappa du dv'
\]

where

\[
f(q, q') = \exp [-\sigma^2(k_{sx}^2 + k_{sy}^2 + q^2 + q'^2 - k_{sx}(q + q') + k_{sy}(q + q'))
\]

\[
f_1(q) = \exp [-\sigma^2(k_{sx} - q)(k_{sy} + q) \rho_1(\tau + \xi - \xi', \kappa + \zeta - \zeta')]
\]

\[
f_2(q, q') = \exp [\sigma^2(k_{sx} - q)(k_{sx} + k_{sy} + q') \rho_2(\tau, \kappa)]
\]

\[
f_3(q, q') = \exp [\sigma^2(k_{sx} - q)(k_{sx} + q') \rho_3(\xi + \tau, \kappa + \zeta)]
\]

\[
f_4(q, q') = \exp [\sigma^2(k_{sy} + q)(k_{sy} + q') \rho_4(\xi', \zeta - \kappa)]
\]

\[
f_5(q, q') = \exp [-\sigma^2(k_{sx} - q')(k_{sx} + q') \rho_5(\tau, \kappa)]
\]

The above equations give the complete representation of the bistatic scattering coefficient for a randomly rough surface. Consider first \( \sigma_{qp}^k(s) \). The exponential factor involving the surface correlation function in the integrand of (26) can be expressed as a sum of two terms,

\[
e^{\sigma^2(k_{sx} + k_{sy})^2 \rho(\xi, \zeta)} = 1 + \sum_{n=1}^{\infty} \left[ \sigma^2(k_{sx} + k_{sy})^2 \right]^n \rho^n(\xi, \zeta) \frac{n!}{n!}
\]

Substituting this into the integrand of (26) we have

\[
\sigma_{qp}^k(s) = \frac{k^2}{4\pi} \left| f_{qp} \right|^2 e^{-\sigma^2(k_{sx} + k_{sy})^2} \int \left[ e^{\sigma^2(k_{sx} + k_{sy})^2 \rho(\xi, \zeta)} - 1 \right] \cdot e^{j(k_{sx} - k_{sx}) \xi + (k_{sy} - k_{sy}) \zeta} d\xi d\zeta
\]

\[
= \frac{k^2}{2} \left| f_{qp} \right|^2 e^{-\sigma^2(k_{sx} + k_{sy})^2} \cdot \sum_{n=1}^{\infty} \left[ \sigma^2(k_{sx} + k_{sy})^2 \right]^n \frac{W^n(\xi, \zeta)}{n!}
\]

(30)

where the amplitude of Kirchhoff terms is found from

\[
\frac{4\pi R^2 \cdot P^k_{qp}}{E_o A_o} = \frac{4\pi R^2 \cdot (CE_o)^2}{E_o A_o} = \frac{4\pi R^2 \cdot E_o^2}{E_o A_o} \cdot \frac{k^2}{(4\pi R)^2} = \frac{k^2}{4\pi}
\]

(31)

and \( W^n(\xi, \zeta) \) is the roughness spectrum of the surface related to the \( nth \) power of the
Gaussian surface correlation function by the Fourier transform as follows:

\[ W^m(k_{xx}, k_{yy}, k) = \frac{1}{2\pi} \int \rho^m(\xi, \zeta) \cdot e^{j(k_{xx}, \xi, k_{yy}, \zeta) - j(k_{xx}, k_{yy}, k, \xi, \zeta)} d\xi d\zeta \]  

(32)

Note that (30) is the Kirchhoff scattering coefficient. It accounts for single scattering only. This is evidenced by the fact that, although (30) contains an infinite sum, only one pair of surface spectral components appears in the scattering process.

Next, we consider the cross term, \( \sigma_{ap}^{kc}(s) \). From (27) the factors that contain the surface-height autocorrelation functions in the integrand can again be written as a sum of two terms as in (29). Then, the following factor in the integrand is rewritten as

\[
\begin{align*}
&= \sum_{n=0}^{\infty} \left[ \frac{\sigma^2(k_x + q)(k_x - q)\rho_1(\xi - \xi', \zeta - \zeta')}{i!} - 1 \right] \\
&\quad + \sum_{n=0}^{\infty} \left[ \frac{\sigma^2(k_x + q)(k_x - q)\rho_2(\xi', \zeta')}{j!} - 1 \right] \\
&\quad + \sum_{n=0}^{\infty} \left[ \frac{\sigma^2(k_x + q)(k_x - q)\rho_1(\xi - \xi', \zeta)}{i!} \right] \sum_{m=1}^{\infty} \left[ \frac{\sigma^2(k_x + q)(k_x - q)\rho_3(\xi', \zeta')}{j!} \right] \\
&\quad + \sum_{n=0}^{\infty} \left[ \frac{\sigma^2(k_x + q)(k_x - q)\rho_2(\xi', \zeta)}{j!} \right] \sum_{m=1}^{\infty} \left[ \frac{\sigma^2(k_x + q)(k_x - q)\rho_3(\xi', \zeta')}{j!} \right] \\
&\quad + \sum_{n=0}^{\infty} \left[ \frac{\sigma^2(k_x + q)(k_x - q)\rho_1(\xi - \xi', \zeta)}{i!} \right] \sum_{m=1}^{\infty} \left[ \frac{\sigma^2(k_x + q)(k_x - q)\rho_3(\xi', \zeta)}{j!} \right] \\
&\quad + \sum_{n=0}^{\infty} \left[ \frac{\sigma^2(k_x + q)(k_x - q)\rho_2(\xi', \zeta)}{j!} \right] \sum_{m=1}^{\infty} \left[ \frac{\sigma^2(k_x + q)(k_x - q)\rho_3(\xi', \zeta)}{j!} \right] \\
&\quad \sum_{n=0}^{\infty} \left[ \frac{\sigma^2(k_x + q)(k_x - q)\rho_1(\xi - \xi', \zeta - \zeta')}{i!} \right] \\
&\quad \sum_{m=1}^{\infty} \left[ \frac{\sigma^2(k_x + q)(k_x - q)\rho_3(\xi', \zeta - \zeta')}{j!} \right]
\end{align*}
\]

In (33) we have omitted the arguments in the surface correlation function by adding subscripts to keep track of changes in arguments. Of the above six terms, we expect the first two and the fifth one to be important since the other three are multiplied by a sum of terms with alternating signs, \( \sum_{n=1}^{\infty} \left[ -\sigma^2(k_x + q)(k_x - q)\rho_1(\xi - \xi', \zeta - \zeta') \right] \), whose magnitude is less than one. Thus, we approximate this scattering coefficient by three terms as

\[
\sigma_{ap}^{kc}(s) = \left( \frac{1}{2} \right) \frac{k^2}{16\pi} \Re \left[ \int f_{ap}^{*} \int f_{ap}(-k_x, -k_y) \right] \cdot e^{-\sigma^2(k_x + q + k_x + q - k_x + k_x)} \cdot \int e^{\sigma^2(k_x + q)(k_x - q)\rho_1(\xi - \xi', \zeta - \zeta')} \cdot \left[ \int e^{-\sigma^2(k_x + q)(k_x - q)\rho_1(\xi - \xi', \zeta - \zeta')} \cdot \int e^{-\sigma^2(k_x + q)(k_x - q)\rho_1(\xi - \xi', \zeta - \zeta')} \cdot d\xi d\zeta d\xi' d\zeta'
\]

\[
\int e^{j(k_{xx}, \xi, k_{yy}, \zeta) - j(k_{xx}, \zeta, k_{yy}, \xi)} d\xi d\zeta d\xi' d\zeta'
\]
\[
+ \left( \frac{1}{2} \right) \frac{k^2}{16\pi} \int e^{-\sigma^2(k_x + k_x + k_{x'} + q^2 + k_{x'} - k_{x'})} d\xi d\eta d\zeta d\zeta' d\zeta'' d\zeta''' \]

The method of evaluation of this scattering coefficient is based upon the properties of the Fourier transform of the delta function.

\[
\delta(t) \cdot e^{-i\omega t} dt = 1
\]  

(35)

\[
\int f(u) \cdot \delta(u - k_x) du = f(k_x)
\]

(36)

and

\[
\frac{1}{2\pi} \int e^{i\omega t} d\omega = \delta(t)
\]

(37)

With the interchange of \( t \) and \( \omega \), the integration becomes

\[
\frac{1}{2\pi} \int e^{i\sigma t} dt = \delta(\omega)
\]

(38)

The scattering coefficient of cross term becomes
\[ \sigma_{\text{sc}}(s) = \frac{k^2}{32\pi} \text{Re}\{\int [f_{\text{sc}}^* F_{\text{sc}}(-k_x, -k_y)] \cdot \]

\[ \int \int [k(q)k_1(q)[k_z(q)k_3(q) - 1] + k(q)k_1(q)[k_z(q)k_3(q) - 1]] e^{i k_z \xi + j k_y \zeta + j \mu (\xi - \xi') + j \nu (\zeta - \zeta')} d\xi d\zeta d\xi' d\zeta' d\xi' d\zeta' d\xi' d\zeta' d\xi' d\zeta' d\xi' d\zeta'} \]

\[ (39) \]

where

\[ k(q) = \exp[-\sigma^2(k_{sz}^2 + k_z^2 + k_z + q^2 - q k_z + q z)] \]

\[ k_1(q) = \exp[-\sigma^2(k_{sz} k_z - q^2 + q k_z - q k_z) \rho_1(\xi - \xi', \zeta - \zeta')] \]

\[ k_2(q) = \exp[\sigma^2(k_{sz} - q)(k_z + k_z) \rho_2(\xi', \zeta)] \]

\[ k_3(q) = \exp[\sigma^2(k_z + q)(k_z + k_z) \rho_3(\xi', \zeta')] \]

The single scattering coefficient including the single sum terms in the above is similar in character to the Kirchhoff term. The double sum term requires integration between pairs of surface spectral components indicating interaction between surface spectral components and hence represents multiple scattering.

Finally, we consider the third term in the scattering coefficient. To evaluate the integral in it we rewrite the factor which contains the surface-height autocorrelation function as follows:

\[ \exp[-\sigma^2(k_{sz} - q)(k_z + q) \rho_6(\tau + \xi - \xi', \kappa + \zeta - \zeta')] \]

\[ -\sigma^2(k_{sz} - q')(k_z + q') \rho_1(\tau, \kappa) \{ \exp[\sigma^2(k_{sz} - q)(k_{sz} - q') \rho_2(\xi, \zeta)] \] \[ + \sigma^2(k_z + q')(k_z + q') \rho_5(\xi', \zeta') + \sigma^2(k_{sz} - q)(k_z + q') \rho_3(\xi + \tau, \kappa + \zeta) \]

\[ + \sigma^2(k_{sz} - q')(k_z + q) \rho_4(\xi' - \tau, \zeta' - \kappa) - 1 \}

\[ = \sum_{p=1}^{\infty} \frac{[-\sigma^2(k_{sz} - q)(k_z + q') \rho_1(\tau, \kappa)]^p}{p!} \]

\[ \sum_{q=1}^{\infty} \frac{[-\sigma^2(k_{sz} - q)(k_z + q) \rho_6(\tau + \xi - \xi', \kappa + \zeta - \zeta')]}{q!} \]

\[ \sum_{n=1}^{\infty} \frac{[\sigma^2(k_{sz} - q)(k_z + q) \rho_3(\xi + \tau, \kappa + \zeta)]^n}{n!} \]

\[ \sum_{l=1}^{\infty} \frac{[\sigma^2(k_{sz} - q)(k_z + q) \rho_4(\xi' - \tau, \zeta' - \kappa)]^l}{l!} \]

\[ \sum_{i=1}^{\infty} \frac{[\sigma^2(k_z + q')(k_z + q) \rho_5(\xi', \zeta')]^i}{i!} \sum_{m=1}^{\infty} \frac{[\sigma^2(k_{sz} - q)(k_{sz} - q') \rho_2(\xi, \zeta)]^m}{m!} - 1 \]
where

\[
\begin{align*}
\sigma_{np} & \approx \frac{k^2}{32} \left\{ F_{ap}(-k_x, -k_x) \right\}^2 [f(q, q') \tilde{f}_2^m(q, q') + f(q, -q') \tilde{f}_2^m(q, -q') \\
& + f(-q, q') \tilde{f}_2^m(-q, -q') + f(-q, -q') \tilde{f}_2^m(-q, -q')] \\
& + F_{ap}(-k_x, -k_x)F_{ap}^*(-k_x, -k_x)[f(q, q') \tilde{f}_2^m(q, q') + f(q, -q') \tilde{f}_2^m(q, -q') \\
& + f(-q, q') \tilde{f}_2^m(-q, q') + f(-q, -q') \tilde{f}_2^m(-q, -q')] \\
& + \frac{1}{2\pi} \int [f_{uv2}^m(q, q')f_{uv5}^n(q, q')f(q, q') + f_{uv2}^m(q, -q')f_{uv5}^n(q, -q')f(q, -q')] \\
& + f_{uv2}^m(-q, q')f_{uv5}^n(-q, q')f(-q, -q') + f_{uv2}^m(-q, -q')f_{uv5}^n(-q, -q')f(-q, -q')]dudv \\
& + \frac{1}{2\pi} \int \{ F_{ap}(u, v)F_{ap}^*(-u - k_x, -v - k_y)[f_{uv3}^n(q, q')f_{uv4}^m(q, q')f(q, q') \\
& + f_{uv3}^n(-q, q')f_{uv4}^m(-q, q')f(q, -q') + f_{uv3}^n(-q, -q')f_{uv4}^m(-q, -q')f(-q, -q') \\
& + f_{uv3}^n(-q, -q')f_{uv4}^m(-q, -q')f(-q, -q')]dudv \}
\end{align*}
\]

There are sixty terms in the above expression. Terms involving more than one sum represent multiple scattering. If the surface deviation, \(k\sigma\), is small or the surface rms slope is small, only terms involving a single sum are important. If surface deviation, \(k\sigma\), becomes large, the multiple scattering terms involving more than two sums makes the major contribution to the bistatic scattering. The evaluation of scattering coefficient is based upon the properties of the Fourier transform of the delta function. The complementary scattering coefficient becomes

\[
\begin{align*}
\tilde{f}_2^m(q, q') &= \sum_{m=1}^{\infty} \frac{[\sigma^2(k_x - q)(k_x - q')]}{m!} W^m(k_x - k_x, k_y - k_y) \\
\tilde{f}_2^m(q, q') &= \sum_{n=1}^{\infty} \frac{[\sigma^2(k_x - q)(k_x + q')]}{n!} W^n(k_x - k_x, k_y - k_y)
\end{align*}
\]
\[ f_4^n(q, q') = \sum_{m=1}^{\infty} \frac{[\sigma^2 (k_{sz} - q')(k_z + q)]^m}{m!} W^n (k_{sx} - k_x, k_{sy} - k_y) \]
\[ f_5^n(q, q') = \sum_{m=1}^{\infty} \frac{[\sigma^2 (k_z + q') (k_z + q)]^m}{m!} W^n (k_{sx} - k_x, k_{sy} - k_y) \]
\[ f_{w2}^m(q, q') = \sum_{n=1}^{\infty} \frac{[\sigma^2 (k_{sz} - q)(k_{sz} - q')]^m}{n!} W^n (k_{sx} + u, k_{sy} + v) \]
\[ f_{w3}^m(q, q') = \sum_{n=1}^{\infty} \frac{[\sigma^2 (k_{sz} - q)(k_z + q')]^m}{n!} W^n (k_{sx} + u, k_{sy} + v) \]
\[ f_{w4}^m(q, q') = \sum_{n=1}^{\infty} \frac{[\sigma^2 (k_{sz} - q)(k_z + q')]^m}{n!} W^n (k_z + u, k_{sy} + v) \]
\[ f_{w5}^n(q, q') = \sum_{n=1}^{\infty} \frac{[\sigma^2 (k_z + q')(k_z + q)]^m}{n!} W^n (k_z + u, k_z + v) \]

(42)

The Kirchhoff and the complementary field coefficients \( f_{aq} \) and \( F_{aq} \) in (34), (41) and (42) are given in appendix of [3]. The sum of these three equations modified by the shadowing function is the final representation of the scattering coefficient for moderately rough surfaces.

### Single and multiple scattering coefficient

Finally we split the scattering coefficient into two terms: single scattering and multiple scattering coefficients. In (42) the single scattering terms are represented by terms with only one sum and do not involve \( u, v \) integration, while terms with more than one sum and \( u, v \) integration represent multiple scattering. It is interesting to note that (42) is also valid for large \( k\sigma \) values, because theoretically the series expansion of exponential factors does not require \( k\sigma \) to be small. Thus, if we let \( k\sigma \) approach infinity, we see that the single scattering terms vanish and only the multiple scattering terms remain finite.

The single scattering coefficient from the summation of (34), (41) and (42) is

\[ \sigma_{aq} = \frac{k^2}{2} e^{-\sigma^2 (k_{sz} + k_z)} \sum_{n=1}^{\infty} \sigma^{2n} |\tilde{f}_{aq}|^2 \frac{W^n (k_{sx} - k_x, k_{sy} - k_y)}{n!} \]

(43)

where

\[
|\tilde{f}_{aq}|^2 = (k_{sz} + k_z)^{2n} |f_{aq}|^2 e^{-2\sigma^2 k_{sz} k_z}
\]
\[
+ \frac{1}{2} \left[ \sum_{n=1}^{\infty} [\sigma^2 (k_{sz} - q)(k_{sz} + k_z)]^n + [\sigma^2 (k_z + q)(k_{sz} + k_z)]^n \right] f_{aq} \cdot F_{aq} (-k_x, -k_y)
\]
\[
+ \frac{1}{2} \left[ \sum_{n=1}^{\infty} [\sigma^2 (k_{sz} - q)(k_{sz} + k_z)]^n + [\sigma^2 (k_z + q)(k_{sz} + k_z)]^n \right] f_{aq} \cdot F_{aq} (-k_{sx}, -k_{sy})
\]
\[
+ \frac{1}{16} \left[ \sum_{n=1}^{\infty} [\sigma^2 (k_{sz} - q)(k_{sz} - q')]^n h(q, q') + [\sigma^2 (k_z + q)(k_{sz} + q')]^n h(q, q') \right]
\]
\[
+ \left[ (k_{sz} + q)(k_{sz} - q') \right]^n h(-q, q') + [(k_z + q)(k_{sz} + q')]^n h(-q, -q')
\]
\[
+ F_{aq} (-k_x, -k_y) F_{aq} (-k_{sx}, -k_{sy}) \left[ [\sigma^2 (k_{sz} - q)(k_z + q')]^n h(q, q') + [\sigma^2 (k_{sz} - q)(k_z - q')]^n h(q, -q') \right]
\]
\[
h(q, -q') + [(k_{sz} + q)(k_z + q')]^n h(-q, q') + [(k_{sz} + q)(k_z - q')]^n h(-q, -q')
\]
+ F_{q'p}^\ast (-k_x, -k_y) F_{q'p}(-k_{sx}, -k_{sy}) \{ [(k_z + q)(k_{z'} - q')]^n h(q, q') + [(k_z + q)(k_{z'} + q')]^n h(q, -q') \\
+ [(k_z - q)(k_{z'} - q')]^n h(-q, q') + [(k_z - q)(k_{z'} + q')]^n h(-q, -q') \}

where

\[ r_i(q) = \exp[-\sigma^2(k_{sx}^2 + k_{sy}^2 - k_{z'} q + k_z q)] \]
\[ h(q, q') = \exp[-\sigma^2(q^2 + q'^2 - k_{z'}(q + q') + k_z(q + q'))] \]

From (26) and (28) the scattering coefficient representing multiple scattering is given by the following terms:

\[
\sigma_{q,p}^m = \frac{k^2}{8\pi} \text{Re} \left\{ f_{q,p}^\ast \int F_{q,p}(u,v) \left[ k(q) f_{av1}^m(q) f_{aw2}^m(q) + k(-q) f_{aw1}^m(-q) f_{aw6}^m(-q) \right] dudv \right\} + \frac{k^2}{64\pi} \int \left\{ \left| f_{q,p}(u,v) \right|^2 \left[ f(q, q') f_{av2}^m(q) f_{aw6}^m(q) + f(q, -q') f_{aw2}^m(-q) f_{aw6}^m(-q) \right] \right. \\
+ f(-q, q') f_{aw2}^m(-q, q') f_{aw5}^m(-q, q') + f(-q, -q') f_{aw2}^m(-q, -q') f_{aw5}^m(-q, -q') \\
+ f_{q,p}(u,v) f_{q,p}^\ast (-u - k_x - k_{sx}, -v - k_y - k_{sy}) [ f(q, q') f_{aw3}^m(q, q') f_{aw4}^m(q, q') + \\
\left. + f(-q, q') f_{aw3}^m(-q, -q') f_{aw4}^m(-q, -q') \right] dudv \right\}
\]

where

\[ k(q) = \exp[-\sigma^2(k_{sx}^2 + k_{sy}^2 + k_{z}^2 + q^2 - k_{z'} q + k_z q)] \]
\[ f(q, q') = \exp[-\sigma^2(k_{sx}^2 + k_{sy}^2 + q^2 + q'^2 - k_{z'}(q + q') + k_z(q + q'))] \]

\[
\begin{align*}
    f_{av1}^m(q) &= \sum_{n=1}^{\infty} \frac{\sigma^2(k_z - q)(k_{z'} - q')^m}{n!} W^n(k_{sx} + u, k_{sy} + v) \\
    f_{av2}^m(q, q') &= \sum_{m=1}^{\infty} \frac{\sigma^2(k_z - q)(k_{z'} - q')^m}{m!} W^n(k_{sx} + u, k_{sy} + v) \\
    f_{av3}^m(q, q') &= \sum_{m=1}^{\infty} \frac{\sigma^2(k_z + q)(k_{z'} + q')^m}{m!} W^n(k_{sx} + u, k_{sy} + v) \\
    f_{av4}^m(q, q') &= \sum_{m=1}^{\infty} \frac{\sigma^2(k_z + q)(k_{z'} + q')^m}{n!} W^n(k_{sx} + u, k_{sy} + v) \\
    f_{av5}^m(q, q') &= \sum_{m=1}^{\infty} \frac{\sigma^2(k_z + q)(k_{z'} + q')^m}{n!} W^n(k_{sx} + u, k_{sy} + v) \\
    f_{av6}^m(q) &= \sum_{m=1}^{\infty} \frac{\sigma^2(k_z + q)(k_{z'} + q')^m}{m!} W^n(k_{sx} + u, k_{sy} + v)
\end{align*}
\]
Shadowing functions for single and multiple scattering

Due to the use of approximate tangential surface fields in the derivative of all the scattering coefficients in the Integral Equation Model, the shadowing functions are required to modify the scattering coefficients from rough surfaces. In single scattering we need to correct the scattering coefficients with shadowing functions separately by multiplying the final average scattered power a shadowing function. In multiple scattering calculation the problem is much more complex. We need to separate the scattered field into the upward and downward scattered fields (Figure 1).

\[
\begin{align*}
\text{Incident} & \quad \theta_i \\
(1-\cos) \text{upward}_1 & \quad \theta_s \\
\text{upward}_2 & \quad (1-\cos) \text{upward}_1 \\
\text{downward}_1 & \quad \theta_s
\end{align*}
\]

Figure 1. The upward scattering and downward scattering geometry in single and multiple scattering with shadowing function.

The upward and downward multiple scattering coefficient should be identified, defined and modified by the shadowing function. The incident angle always lies between 0 and 90 degrees, however the scattered angle may lie between 0 and 180 degrees. The scattering coefficient can be split into two parts in terms of the scattering angles: the upward scattering \((0^\circ \leq \theta_s < 90^\circ)\) and the downward scattering \((90^\circ \leq \theta_s \leq 180^\circ)\). The fraction of the upward scattering intercepted by the surface roughness is \(1 - S(\theta_s)\). The fraction that leaves the rough interface in upward direction is \(S(\theta_s)\). As shown in figure 1 the downward scattered waves must be intercepted by the rough interface and can be regarded as the second incoming wave intercepted by rough surfaces in the multiple scattering process.

The explicit focus of the shadowing functions for the incident and scattered waves are

1. In single scattering the shadowing function for the incident waves depends upon the cotangent of the incident angle. The incident shadowing function \(s(\theta_i, \sigma_s)\) is

\[
s(\theta_i, \sigma_s) = [1 - \frac{1}{2} \text{erfc}\left(\frac{\cot \theta_i}{\sigma_s \sqrt{2}}\right)] [1 + f(\theta_i, \sigma_s)]^{-1}
\]

where

\[
f(\theta_i, \sigma_s) = \frac{1}{2} \left\{ \frac{2}{\pi} \frac{\sigma_s \sqrt{2}}{\cot \theta_i} \exp\left(-\frac{\cot \theta_i^2}{2\sigma_s^2}\right) - \text{erfc}\left(\frac{\cot \theta_i}{\sigma_s \sqrt{2}}\right) \right\}
\]

(2) In single scattering the shadowing function for the scatter waves depends upon the cotangent
of the scatter angle, \( \cot \theta_s \). The scatter shadowing function \( s(\theta_s, \sigma_s) \) is

\[
s(\theta_s, \sigma_s) = [1 - \frac{1}{2} \text{erfc}(\frac{\cot \theta_s}{\sigma_s \sqrt{2}})][1 + f(\theta_s, \sigma_s)]^{-1}
\]

(47)

where

\[
f(\theta_s, \sigma_s) = \frac{1}{2} \sqrt{\frac{2}{\pi}} \frac{\sigma_s \sqrt{2}}{\cot \theta_s} \exp(-\frac{\cot^2 \theta_s}{2\sigma_s^2}) - \text{erfc}(\frac{\cot \theta_s}{\sigma_s \sqrt{2}})
\]

(48)

The rescattered shadowing function \( s(\theta, \sigma_s) \) inside the integration is expressed as

\[
s(\theta, \sigma_s) = s(x, \sigma_s) = [1 - \frac{1}{2} \text{erfc}(\frac{\cot \theta}{\sigma_s \sqrt{2}})][1 + f(\theta, \sigma_s)]^{-1}
\]

(49)

where

\[
f(\theta, \sigma_s) = \frac{1}{2} \sqrt{\frac{2}{\pi}} \frac{\sigma_s \sqrt{2}}{\cot \theta} \exp(-\frac{\cot^2 \theta}{2\sigma_s^2}) - \text{erfc}(\frac{\cot \theta}{\sigma_s \sqrt{2}})
\]

(50)

In multiple scattering the shadowing function depends upon the cotangent of the incident angle \( \theta_p \) of the rescattered field, i.e.,

\[
\cot \theta_p = \frac{1}{\sqrt{x^2 - 1}}
\]

(51)

where

\[
u = r \cos \delta
\]

\[
v = r \sin \delta
\]

(52)

and \( x \) is the normalized value of \( r \), i.e.

\[
x = \frac{r}{k}
\]

(53)

The different \( u \) and \( v \) value of the shadowing function in multiple scattering represents different scattering directions.

**MODEL PREDICTION**

A set of plots for the like- and cross-polarized bistatic scattering coefficient versus the azimuth angle are shown in figure 2 through 5 respectively. In the figures the incident angle is 45° and scatter polar angle is 20°. Zero value of the azimuth angle corresponds to the forward scattering
and azimuth value of 180° represents the backscatter. For the plane orthogonal to the plane of incidence the azimuth angle is 90°. The prediction strength of electromagnetic wave is from the statistically rough surface with normalized surface correlation length 1.0 and different surface rms height. From figure 2 through 5 we have the strongest scatter power in the specular direction for the like-polarized scattering, but there is a maximum scatter power in the direction orthogonal to the plane of incidence for the cross-polarized scattering. From the prediction for the electromagnetic wave scattering from random rough surfaces in this paper we found the single scattering is the major contribution for the electromagnetic wave scattering from rough surfaces with small surface rms height and rms surface slope.

Figure 2 through 5 show the comparisons of an azimuth slice of the bistatic scattering pattern with different normalized surface height. The incidence angle is 45° and the scattering angle is 20°. The normalized surface heights to the incidence wave number are 0.2, 0.3 and 0.4 respectively. The normalized surface correlation length is 1.0. The rms surface slopes are 0.283, 0.4242 and 0.5656 respectively. The azimuth pattern exhibits a strong lobe along the specular direction for the like-polarization components, but the cross-polarized scattering component exhibits a local null along the specular direction in figure 2 through 5. On the contrary, the azimuth pattern exhibits a strong lobe along the orthogonal plane of incidence for the cross-polarization components. The cross-polarized scattering component exhibits a local null along the plane orthogonal to the plane of incidence. Those satisfies the scattering properties that the single scattering makes a major contribution along the specular direction and multiple scattering makes a major contribution along the plane orthogonal to the plane of incidence. For surfaces with moderate surface height and rms slope the single scattering along the specular direction is important in like polarized and multiple scattering contributions becomes negligible. Multiple scattering is the major source for cross polarization along the plane orthogonal to the plane of incidence. The scatter pattern of VH polarization is very similar to that of HV polarization and the scatter pattern of VV polarization is very similar to that of HH polarization. In VV-polarized scattering the maximum scatter power reaches in the forward scattering direction, but the maximum HH-polarized scattering coefficient takes place in the backscattering direction as our expectation. Further the VV-polarized scattering coefficient is strongly dependent on the surface parameters. In cross-polarized scattering the HV-polarized scattering coefficient is strongly dependent on the surface parameters. The VH-polarized scattering coefficient is independent of the surface parameters except in the plane orthogonal to the plane of incidence. As the expectation the scattering coefficient increases for the larger normalized surface height and rms surface slope. Figures 6 through 9 show the dB value of bistatic scattering strength along the different azimuth angle for the comparisons with the measured data. Figures 10 through 12 show the comparisons of like- and cross-polarized scattering from random rough surfaces. Figures 10 through 12 show the scattering strength is independent of the surface parameters for the surface slope of 0.283, the VV and HH like-polarized scattering are very similar and VH and HV cross-polarized scattering strength are same. The figures reveal that the electromagnetic waves scattered out of the plane of incidence are strongly dependent on the polarization orientation of the incident electric field from the random rough surfaces with rms surface slope larger than 0.4.
Figure 2. Like VV-polarized electromagnetic wave scattering from rough surfaces with different normalized surface rms height along the different azimuth angle. The normalized surface correlation length is 1.0. The incident angle is $45^\circ$ and scatter angle is $20^\circ$.

Figure 3. Like HH-polarized electromagnetic wave scattering from rough surfaces with different normalized surface rms height along the different azimuth angle. The normalized surface correlation length is 1.0. The incident angle is $45^\circ$ and scatter angle is $20^\circ$.

Figure 4. Cross VH-polarized electromagnetic wave scattering from rough surfaces with different normalized surface rms height along the different azimuth angle. The normalized surface correlation length is 1.0. The incident angle is $45^\circ$ and scatter angle is $20^\circ$. 
Figure 5. Cross HV-polarized electromagnetic wave scattering from rough surfaces with different normalized surface rms height along the different azimuth angle. The normalized surface correlation length is 1.0. The incident angle is 45° and scatter angle is 20°.

Figure 6. Cross HV-polarized electromagnetic wave scattering from rough surfaces with different normalized surface rms height along the different azimuth angle. The normalized surface correlation length is 1.0. The incident angle is 45° and scatter angle is 20°.

Figure 7. Cross VH-polarized electromagnetic wave scattering from rough surfaces with different normalized surface rms height along the different azimuth angle. The normalized surface correlation length is 1.0. The incident angle is 45° and scatter angle is 20°.
Figure 8. Like HH-polarized electromagnetic wave scattering from rough surfaces with different normalized surface rms height along the different azimuth angle. The normalized surface correlation length is 1.0. The incident angle is $45^\circ$ and scatter angle is $20^\circ$.

Figure 9. Like VV-polarized electromagnetic wave scattering from rough surfaces with different normalized surface rms height along the different azimuth angle. The normalized surface correlation length is 1.0. The incident angle is $45^\circ$ and scatter angle is $20^\circ$.

Figure 10. Like- and cross-polarized electromagnetic wave scattering from rough surfaces with normalized surface rms height 0.4 and normalized surface correlation length 1.0 along the different azimuth angle. The incident angle is $45^\circ$ and scatter angle is $20^\circ$. 
As an illustration we compared the model prediction with the types of backscatter measurements made by B. Hauck et al. [1]. Figure 13 accounts for the like-polarized incoherent scattering from rough surfaces with surface rms height 0.2 and correlation length 1.0 normalized to the incident wave number. The surface material is water-soaked form bricks with estimated dielectric constant 63.2. For the isotropic surface being assumed the scattering pattern is symmetric to the plane of incidence. Examination of figure 13 indicates that there is excellent agreement between the model prediction and the measured data [1].

Another three sets of backscattering measurements along the specular direction are used to evaluate the model prediction. Figure 14 shows the comparisons of integral equation model with the measured data for like-polarized backscattering coefficient. These are like-polarized measurements from a perfectly conducting Gaussian distributed surface with normalized rms surface height 0.1 and normalized surface correlation length 1.0. Figure 15 shows the comparisons of integral equation model with the measured data for like-polarized backscattering coefficient.
coefficient from perfectly conducting rough surfaces with normalized surface height 0.1 and normalized correlation length 6.5. Figure 16 shows the comparisons of integral equation model with the measured data for like-polarized backscattering coefficient from perfectly conducting rough surfaces with normalized surface height 1.1, 0.55 and normalized correlation length 3, 1.5, normal to the incident wave number. The operating frequency has a change of factor 2. The measured data reported by Nance’s numerical simulation [1992]. From figure 14, 15 and 16 the excellent trend agrees between Integral equation model and measured data. The difference between the measured data and numerical simulation is within 1 dB.

Figure 13. The comparisons of integral equation model with the measured data for like-polarized backscattering coefficient. The backscattering pattern for random rough surfaces with surface height 0.2 and correlation length 1.0 normal to the incident wave number. The surface material is water-soaked form bricks with estimated dielectric constant 63.2.

Figure 14. The comparisons of integral equation model with the measured data for like-polarized backscattering coefficient. The backscattering pattern for perfectly conducting rough surfaces with normalized surface height 0.1 and normalized correlation length 1.0, normal to the incident wave number.
Figure 15. The comparisons of integral equation model with the measured data for like-polarized backscattering coefficient. The backscattering pattern for perfectly conducting rough surfaces with normalized surface height 0.1 and normalized correlation length 6.5, normal to the incident wave number.

Figure 16. The comparisons of integral equation model with the measured data for like-polarized backscattering coefficient. The backscattering pattern for perfectly conducting rough surfaces with normalized surface height 1.1, 0.55 and normalized correlation length 3, 1.5, normal to the incident wave number. The operating frequency has a change of factor 2.

REFERENCES