A case study on open boundary techniques for electromagnetic field problems with translational symmetry

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Abstract— The paper discusses two finite element open boundary techniques for the solution of electromagnetic field problems with translational symmetry. Firstly, the underlying assumptions and ideas behind the techniques of simple truncation and Kelvin transformation are presented. The techniques are then analyzed computationally with an instructive problem. This consists of a pair of two parallel and long conductive busbars where the force due to currents in opposing directions is to be evaluated. The value of the force calculated analytically is used to check the accuracy of the computed forces. The main features of the numerical models are explained and a comparative study based on the necessary steps to reach a specified accuracy in computed forces is presented. Numerical results show that to obtain accuracy better than 1% using the simple truncation method, very large outer boundaries must be employed. With the Kelvin method, accuracy better than 0.1% is readily achieved without using a large number of mesh elements.

Index Terms— Electromagnetic engineering education, electromagnetic forces, finite element methods.

I. INTRODUCTION

Design and manufacture of electric power equipment is a challenging task and the inter-connection of electrical, mechanical and thermal designs turns out to be a highly specialized process subject to many requirements. In practical design of electrical machines, for example, it is well known that more accurate results can be obtained when thermal and electromagnetic fields are analyzed as coupled fields [1]. In the past three decades, an increasing number of manufacturers rely, to a varied extent, on electromagnetic computer-aided design (CAD) systems in their electrical design processes [2]. The accuracy of electromagnetic field computations plays an important rule in the whole process once results must be extremely accurate if they are to be of any practical use; in many applications the fields must be computed to an accuracy of 1 part in 1000 or better. High accuracy requirements are only satisfied if the materials used in construction of electromagnetic devices are easily measured and characterized, and their properties are reasonably constant. Many other specific questions affect the accuracy of field computations and these involve, for example, the geometrical complexity of the device, magnetic saturation, parasitic effects and the unbounded nature of the fields [3].

In practical engineering applications, many of the electromagnetic field problems are characterized as open domain or open boundary problems. Since the finite element method (FEM) is a finite domain

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method, special techniques to adequately represent the exterior region had to be devised to overcome this difficulty. The several open boundary methods currently available may be separated into two classes: the global methods, which deal with the exterior region as a whole, and the elemental methods where the exterior region is subdivided into a finite number of subregions. The literature documents extensive research into open boundary techniques used in conjunction with the finite element method. Fortunately, there are excellent surveys intended to aid researchers to select one or more techniques suitable for a given application. Almost without exception, researchers working on the finite element simulations of open boundary electromagnetic problems refer back to the early reviews by Emson [4] and Bettess [5]. These reviews describe alternative methodologies which have proved to be more efficient than simple truncation in the analysis of static fields and travelling waves. Of major note is the work of Chen and Konrad [6], a comprehensive survey on the subject. The work makes reference to 158 publications in the context of static and quasi-static electromagnetic field problems. The subject is widely researched, with extensive published theory and many reported applications to real-world engineering [7], [8].

In the present study, two open boundary techniques, namely simple truncation and Kelvin transformation are employed to solve a problem of force calculation. The choice of the test problem or, more specifically, the choice of the design parameter is of crucial importance to assess the performance of different numerical techniques. Some test problems may not be adequate because they deal with very simple field distributions, and the difficulties associated with the modeling of the exterior region, and the consequent numerical errors, may not appear [9]. Computation of forces from approximate numerical field solutions, on the other hand, is a notoriously difficult task, usually subject to large numerical errors and therefore adequate to identify the implications of using different procedures [10]. The open boundary problem considered in this study consists of two parallel and long conductive busbars, where the force due to currents in opposing directions is to be evaluated. For this problem, there is a simple analytical expression for the force between the two busbars and this is used to check the accuracy of different numerical models.

II. LITERATURE REVIEW

Conceptually simple and easily implemented in finite element CAD systems, the method of simple truncation is often taught in introductory finite element CAD simulation courses. Lowther and Freeman [11] have used this method to introduce the concept of exterior problems and show how the increasing distance of the artificial boundary enhances the accuracy of force calculations. More recently, there have appeared publications showing how to increase the accuracy of capacitance calculations of electrostatic devices using either, asymptotic boundary conditions [12] or techniques based on truncation [13].

The Kelvin transformation is one of the spatial transformation schemes in which a physical unbounded domain is converted into a finite domain. This elegant approach has been used by Freeman and Lowther [14] to calculate the characteristic impedance of a simple strip line problem and the energy-based parameters of parallel current-carrying conductors. Soon after, several interesting possibilities of mapping techniques related to the Kelvin transformation, like the idea of “stock” boundaries, have been presented by Lowther, Freeman and Forghani [15]; the study shows the ease with which field quantities are recovered from a database of standard points used to store information related to a previously meshed transformed exterior region.

The Kelvin transformation of a circle cannot be applied directly to axisymmetric and three-dimensional problems. To overcome such a limitation of the technique, Wong and Ciric [16] have identified the necessary steps to develop a more complete formulation applicable to those classes of problems. Their work was further developed by Freeman and Lowther [17], who presented a detailed derivation of the method in the context of a standard finite element CAD system. The ideas of transformation approaches have been taken even further, and new formulations have been developed to accommodate problems in which the outer region might contain both sources and material variation [18]. Recent releases of finite element CAD system known as FEMM [19] have incorporated a procedure to model unbounded axisymmetric problems. Specifically, the added routine called “external region” enables the input of geometrical information related to the semicircles used to define two spheres; one of the spheres represents the region in which the device is located, and the other sphere represents the axisymmetric exterior region. In this “axisymmetric version” of the Kelvin transformation method, the distributions of permittivity and permeability of the interior and mapped exterior regions are related in a way that the same differential equation applies to both regions; as a result, the same solver can be used to analyze the field problem in both regions [20].

III. SIMPLE TRUNCATION AND THE KELVIN TRANSFORMATION

A. *Simple Truncation*

The method is based on the assumption that at a sufficiently far away artificial boundary, the potential or the normal derivative of the potential will be close to zero. It is important to mention two properties of the artificial boundary. Firstly, the introduction of such a boundary at some distance of the device effectively converts the open boundary problem into a closed one by main force. Secondly, the portion of space surrounding the device and extending up to the artificial boundary is important in order to support an accurate estimation of the field in the region where the device is located. While this approach is conceptually simple and easy to realize, highly accurate results are only obtained for sufficiently far away outer boundaries which, in turn, increase memory requirements and CPU time. The simplest rule for locating the artificial boundary is based on the rate of decay of a dipole field, since any device will resemble a dipole at a sufficient distance from its location. The use of the technique implies some judgment on matters of conflicting nature such as the increase in solution accuracy and the reduction of computing requirements. If attempts are made to improve the accuracy of the solution, for example, one may model an excessive large piece of empty space without adding any useful information in the region of actual interest. From the above discussion it becomes clear that the method is computationally expensive.

B. *The Kelvin Transformation*

In the following, the Kelvin transformation method is discussed on the grounds of its basic formulation. The extensions of the method regarding the different forms of representing the outer region and its contents are detailed in [18]. In the classical formulation of the Kelvin transformation, the solution to the open boundary problem is to enclose the region where the device is located with a circular boundary and to create a second circular region. This second circular region has the same radius as the first circle, the same boundary node distribution and only encloses air. The two circular boundaries are connected in a way that the corresponding nodes are joined. This may be performed by applying binary constraints to the node distributions of both circles. The degree of discretization of the two circular regions may be different and, very often, the required number of mesh elements is small in comparison with other methods. This mapping technique is illustrated in Fig. 1.

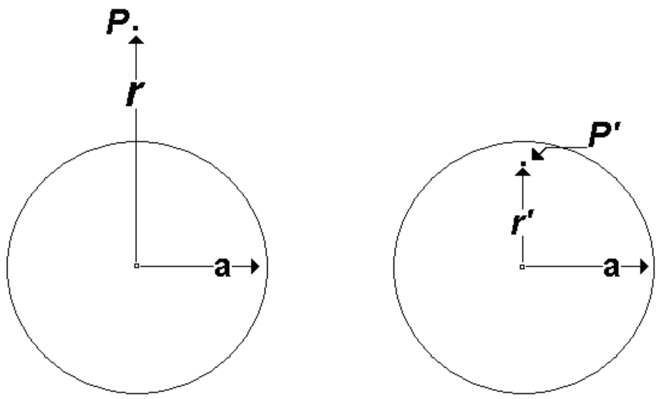

Fig. 1. Schematic representation of the Kelvin transformation.

If a denotes the radius of the two circles, the relationship between the distance r from the centre of the left hand side circle to a generic point P situated at its exterior portion where r>a, and the transformed radius r' is

$$rr' = a^2, \tag{1}$$

where the radius r' represents the distance from the centre of the right hand side circle to the transformed point P'. The second circle exactly represents the space outside the first circle up to infinity. In fact, infinity lies at the centre of this second circle.

IV. TEST PROBLEM

In order to examine the performance of the two open boundary techniques, a problem of force calculation has been chosen. The problem consists of two parallel and long conductive busbars placed in air, where the force due to currents in opposing directions is evaluated. A complete derivation of the analytical expression for the force between two parallel busbars with rectangular shapes and carrying

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currents in opposite directions is presented by Bewley [21]. The problem is presented at the web site of Infolytica Corporation [22].

The geometrical details of the problem are shown in Fig. 2. The bars are 0.2 m apart in the vertical direction, and each bar is 0.1 m high and 0.2 m wide. The current density is 5 A/mm^2, so the currents in the upper and lower bars are equal to +100 kA and −100 kA, respectively.

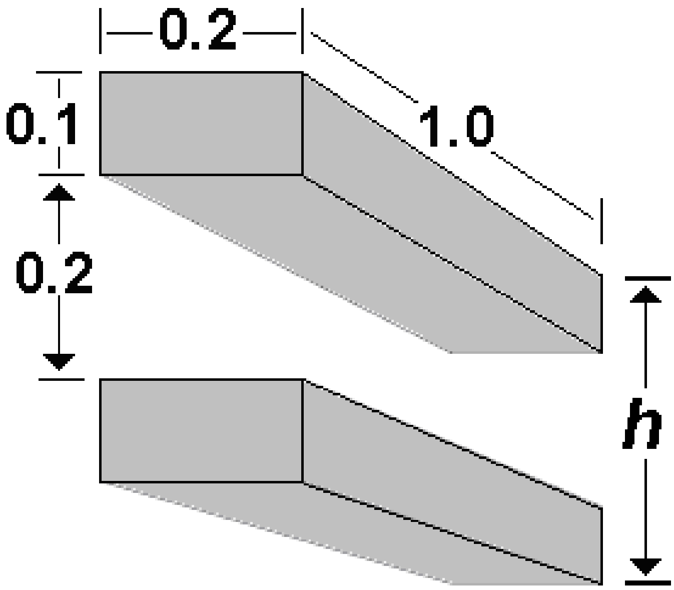

Fig. 2. Parallel rectangular busbar conductors. Dimensions in meter.

The analytical expression for the force per unit length between the two long busbars in free space is

$$F = \frac{2x10^{-7} kI^2}{d} N/m, \tag{2}$$

where k is a constant that depends on the shape of the conductors, and d is the distance between the centers of the busbars, namely 0.3 m. The value of k in this problem is 0.95, so the theoretical prediction for the force density is 6.333 kN/m. The leading dimension in this drawing is the distance h between the top of the upper conductor and the bottom of the lower one. The value used for the device's length in the longitudinal or z direction is 1.0 m; this represents 5 times the width of each busbar and 2.5 times the leading dimension in the x-y plane, so the device may be considered relatively long at its remote end. For 1.0 m depth of the device, the theoretical force is 6.333 kN.

V. IMPLEMENTATION AND TESTS

In order to evaluate the performance of the two techniques, numerical models are built, one for each technique. Numerical field solutions are obtained using the two-dimensional Cartesian magnetostatic processors and solver of the simulation software FEMM [19]. To permit two-dimensional analysis, the set of busbars of Fig. 2 is considered infinitely long and longitudinally uniform. The solutions are obtained for the magnetic vector potential $\boldsymbol{A}$ which in a two-dimensional analysis is assumed entirely z

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directed. It is then possible to work simply with a scalar distribution A which denotes the longitudinal component of the vector $\boldsymbol{A}$. First-order triangular finite elements are employed in all computations. The numerical models involve no unusual or extraordinary geometric difficulties like millimetric gaps or magnetic pole tips, so the mesh artifact is relatively simple. The method employed to control the level of discretization in a given region is based on the specification of a parameter δ known as “mesh size”. This parameter defines a constraint on the largest possible size of the elements’ edges allowed in a given region. The mesh generator Triangle [23] attempts to fill each selected region with nearly equilateral triangles in which the sides are approximately the same length as the specified mesh size parameter.

A. Model for simple truncation

The model consists of a series of square regions, concentric with respect to the device’s centre which has been made to coincide with the origin of the *x-y* plane. Dirichlet boundary conditions are used to simulate the location of a receding boundary, i.e. the modeled domain terminates at that square boundary where the condition A=0 applies. To facilitate the analysis by this method, it is employed a per unit parameter η relating the side of each square boundary and the height h, which is the leading dimension of the physical device in the *x-y* plane. For the sake of simplicity, the parameter η will be referred to as the “boundary size”. Observation of Fig. 2 shows that the distance h is equal to 0.4 m. A cross-sectional drawing of one quarter of the numerical model is shown in Fig. 3.

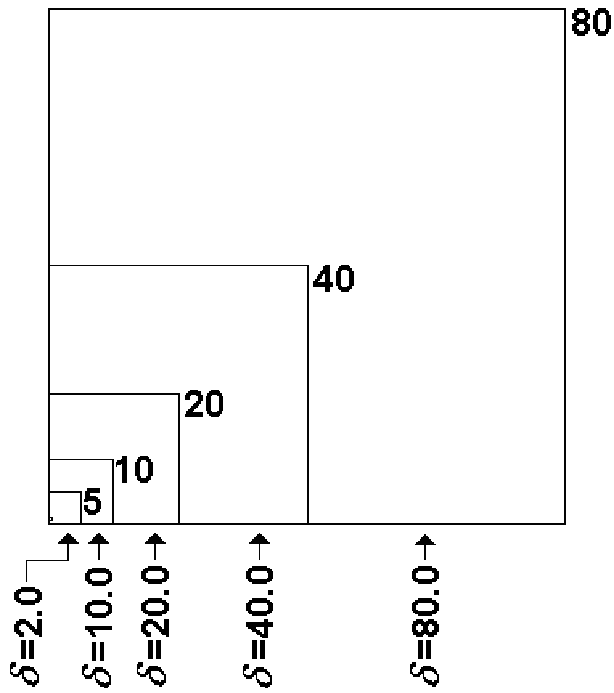

Fig. 3. Model for simple truncation; per unit values of the boundary size η and values in centimeter of the mesh size δ.

The increase in dimensions of the outer boundary follows an arithmetic progression where in each step the side of the outer boundary is doubled and the corresponding area is increased by a factor of 4. In this illustration, per unit values of the boundary size are indicated outside the upper right corner of each square box. Once the outer boundary is moved outward from the central portion of the model, the size of the mesh elements is also increased in the attempt of avoiding meshes with very large number of elements. In the illustration of Fig. 3, the values in centimeter specified for the mesh size δ are indicated at the bottom edge of the drawing.

It is worth here a brief explanation on how to simulate the expansion of the outer boundary of each discretized problem or model configuration. In the first configuration, the innermost concentric region is discretized with mesh size δ=2.0 cm, the condition A=0 is applied to its periphery and no mesh is generated in the other regions. In the second configuration, the first two square regions are discretized with mesh sizes 2.0 and 10.0 cm, the condition A=0 is applied to the periphery of the second region and no mesh is generated in the remaining regions. A similar procedure is used in the creation of the other configurations; the process continues until a specified termination criterion is satisfied.

B. Model for the Kelvin transformation

To use the Kelvin transformation method, the region of prime interest is enclosed by a circular region, and a second air filled circular region is placed alongside. A cross-sectional drawing of the model is shown in Fig. 4. The radii a of both circles are equal to 0.25 m, and the circular region on the right hand side is displaced by 1.5 diameter in the x direction. In this case, each circle is composed by two arcs. The circle on the left hand side is formed by the union of arcs *1* and *2*, whilst the circle on the right hand side is formed by the union of arcs *1'* and *2'*. The corresponding pairs of arcs *1* and *1'* as well as *2* and *2'* should have exactly the same node distributions in order to link together the two domains. In other words, each arc bounding the domain containing the busbars is linked to the corresponding arc in the domain representing the exterior region. Each pair of corresponding arcs is identified or selected, and even periodic boundary conditions are applied to enforce the same value of the magnetic vector potential to the corresponding nodes.

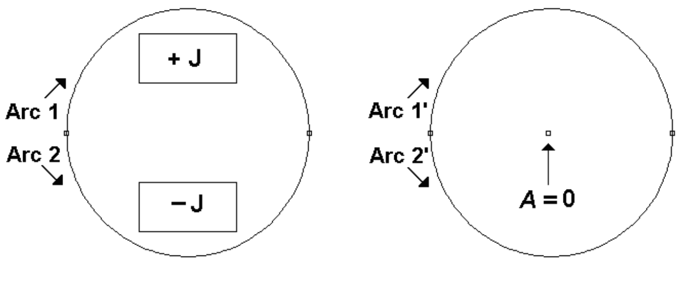

Fig. 4. Circular regions and the Kelvin transformation.

Once the centre of the air-filled domain maps to infinity in the analogous open problem, it is necessary to apply a point property which specifies the condition A=0 and creates a reference point. The definition

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of a reference point eliminates the possibility of numerical difficulties due to the uniqueness of the numerical solution [19].

For the model's discretization, similar meshes are used in both circular regions. For the first configuration or potential solution, the circular regions are discretized with mesh size parameters δ=2.0 cm, the same value used for the first solution by simple truncation.

C. Discretization of the numerical models

The two models are essentially different in their geometrical and topological features. In the truncation method, it is important to place the outer boundary far away from the physical device and this, very often, results in meshes with large numbers of nodal points and mesh elements. With the Kelvin transformation, the increase in radial extent of the circles is unimportant, i.e. the area of the circles need not be increased to produce more accurate numerical solutions. Based on these considerations, it becomes evident that the discretization of the two models can only be compared in a certain sense.

In the illustration presented in Fig. 5, the square region represents the smallest boundary size (η=5) in the analysis by simple truncation. The circle that encloses the pair of busbars in the model for the Kelvin method is shown on the right hand side and represents half of the area to be discretized. Were the Kelvin method to be used the area to be discretized would be 0.39 m^2. With simple truncation, the discretized areas are in the range 4.0–1,024.0 m^2. When compared to the required area to be discretized with the Kelvin method, these areas represent an increase by a factor of nearly 10 for the first configuration and 2,625 for the fifth one. From these figures, it becomes evident that with the Kelvin transformation the area to be discretized and the extent of the finite element mesh can be significantly reduced.

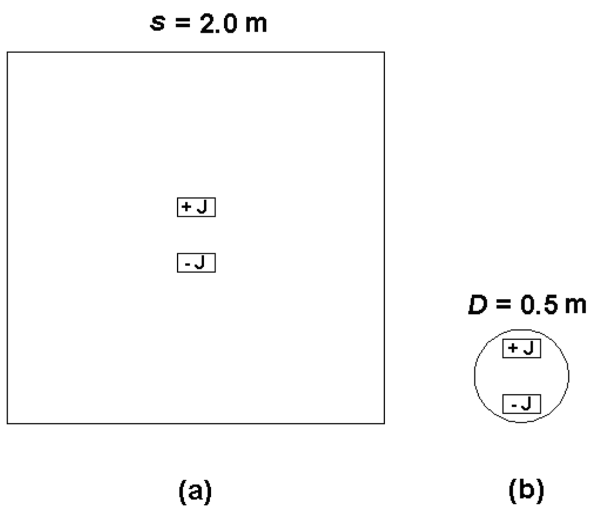

Fig. 5. Outline of the areas to be discretized. (a) Analysis by simple truncation starts with a square boundary for which the side s is 2.0 m long. (b) Analysis by the Kelvin method always uses two circles of 0.5 m diameter.

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VI. NUMERICAL RESULTS

The comparative study is based on the necessary steps to reach accuracy better than 1% in computed forces. The method of weighted Maxwell stress tensor is employed in all force calculations. This method has proved to be one of the most reliable approaches for the calculation of forces from numerical field solutions. Despite the complexity of its mathematical derivation [24], the use of this method in the modern field simulators became easier than that of the classical Maxwell stress tensor method, because no special choice of integration contours is required. The weighted approach represents a modern simulation tool where the contours of integration and the weighting functions are calculated in a completely automated process [25].

The plots presented in Fig. 6 concern the solution of the first and fourth configurations of the model for simple truncation. The plots are scaled to the same view size that displays the entire problem geometry. In each case, it is shown the density plot of the magnetic induction ***B*** in greyscale, together with the equipotential lines. The plot on the left hand side is used to illustrate the truncation of the ***B***-field. The outermost shaded light-grey zone surrounding the pair of conductors includes the portion of space where flux densities lye in the range 14.2─14.8 mT. This zone is suddenly terminated at the upper and lower edges of the drawing where truncation is enforced. In the plot on the right hand side, the shaded zones of magnetic flux density are concentrated in the central portion of the drawing and represent a small percentage of the displayed view size. In fact, these zones can only be observed with the aid of enlarged views. The equipotential lines of this solution present a clear resemblance with those of a dipole field, and this suggests that the outer boundary has been moved far enough out to avoid truncation.

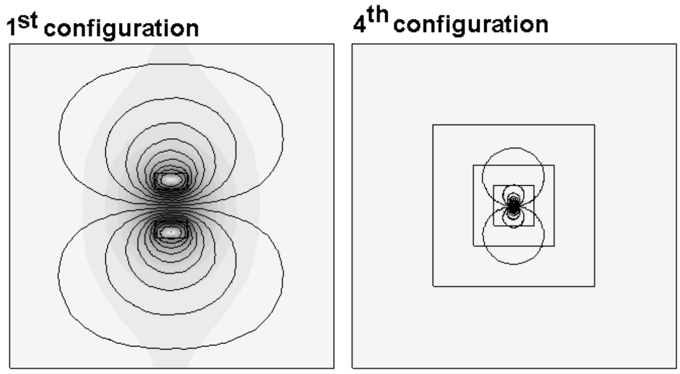

Fig. 6: Density plots of the *B*-field and equipotential lines for simple truncation.

Results of the force computations associated to the simple truncation method are presented in Fig. 7. The circular knots represent the sequence of force computations versus the boundary size. The dashed line represents the analytical value for the force, namely 6.333 kN. The information summarized in Table I concerns the effect of increasing the boundary size on the number of nodes of the finite element meshes, values of computed forces and the corresponding numerical errors.

Observation of the computed force characteristic shows that the use of a boundary very close to the pair of conductors in the first configuration has produced a very low estimate for the force value. A major reduction in force computation error occurs between the first (η=5) and second (η=10) configurations; in this case, the error level has decreased from 8.78 to 2.71 per cent. The required accuracy of the study is only satisfied by the fourth configuration (η=40), where the force computation error is 0.85 per cent. The additional configuration, for which η=80, leads to a very small improvement in the computed force value, despite the significant increase in the area of the modeled domain.

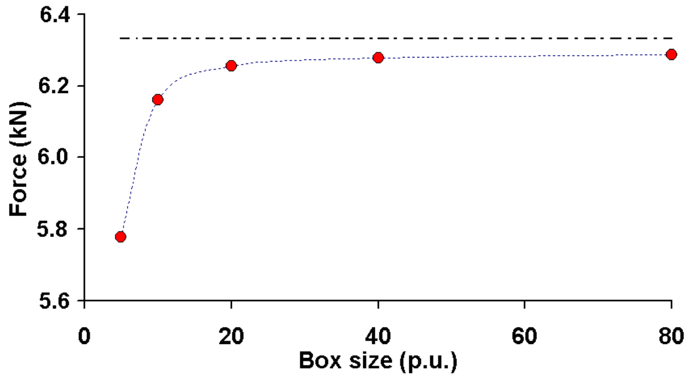

Fig. 7. The effect of increasing the boundary size on the values of computed forces.

TABLE I . THE EFFECT OF INCREASING THE SIZE OF THE SQUARE BOUNDARY.

Boundary size (p.u.)	**Number of nodes**	**Force (kN)**	**Error (%)**
5	2531	5.777	8.78
10	7731	6.162	2.71
20	10645	6.257	1.21
40	13702	6.279	0.85
80	16600	6.287	0.72

A previous analysis of this problem published at the web site of Infolytica Corporation [22] explores two methods of increasing the solution accuracy: adaptive mesh refinement and the increase of the polynomial order of the trial function. Results show that to reach accuracy better than 1% very large boundary sizes must be employed and that, for this particular problem, there is little to choose between those two methods of increasing the solution accuracy; the use of a polynomial order of 3 has produced the same accuracy as that obtained by a combination of polynomial order 2 and adaptive mesh refinement applied to 30% of the elements. The overall implication is that attempts to further decrease the numerical error, from 1% to 0.1% say, would require a considerable computational cost and the use of simulation tools – like polynomial approximating functions of higher order – not provided by every existing finite element CAD system. The Kelvin transformation, on the other hand, offers a way of doing this.

Results concerning the Kelvin transformation method are presented in Fig. 8. The density plot of the magnetic induction ***B*** is shown together with the equipotential lines. A close observation of the left hand side region shows that most flux lines apparently cross out of the region containing the pair of conductors as if unaffected by the presence of any boundary; in fact, these flux lines reappear in the right hand side circular region at the corresponding nodes, complete their flux paths through this air-filled region and then return to the region containing the conductors.

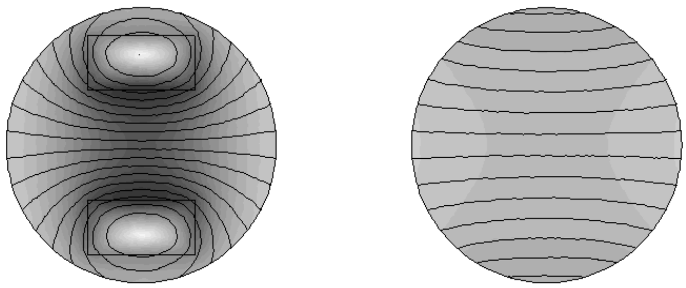

Fig. 8. Density plots of the *B*-field and equipotential lines for the Kelvin transformation method.

The required accuracy of the study is promptly satisfied by the first configuration of the Kelvin transformation model for which the force computation error is only 0.47 per cent. It is worth noting the small number of nodal points and triangular elements of this configuration, namely 1511 and 2730, respectively. These numbers ought to be compared to those used in the first configuration of the truncation approach, namely 2531 nodes and 4923 triangular elements.

In view of the good results produced by the first configuration of the Kelvin method, the task of further reducing the numerical error to obtain accuracy better than 0.1% is then considered. For this, the well known approach of increasing the mesh fineness is employed, and this can be readily done by decreasing the value specified for the mesh size parameter. In the first configuration, the value specified for this parameter is 2.0 cm. For each of the remaining field solutions, the value of this parameter decreases in steps of 0.25 cm, until the new accuracy requirement is satisfied. Results of the force computations associated to the Kelvin transformation method are presented in Fig. 9.

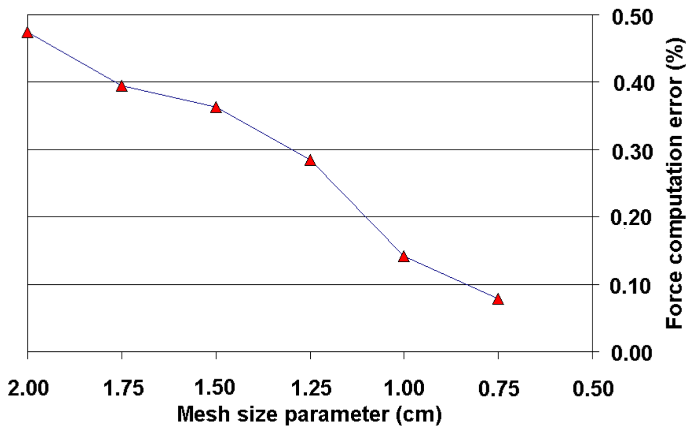

Fig. 9. Force computation error; analysis by the Kelvin transformation method.

In the graph of Fig. 9, the characteristic marked with triangular knots represents the sequence of error in force computations versus the mesh size parameter δ. The information summarized in Table II concerns the effect of decreasing the mesh size parameter on the number of nodes of the finite element meshes, values of computed forces and the corresponding numerical errors.

TABLE II. THE EFFECT OF DECREASING THE MESH SIZE PARAMETER IN THE KELVIN APPROACH.

Mesh size (cm)	Number of nodes	Force (kN)	Error (%)
2.00	1511	6.363	0.47
1.75	1705	6.358	0.39
1.50	2134	6.356	0.36
1.25	2765	6.351	0.28
1.00	4046	6.342	0.14
0.75	7654	6.338	0.08

The results presented in Fig. 9 and Table II demonstrate that the Kelvin transformation method may be used as a means of implicitly moving the outer boundary farther away, without requiring a large number of elements and nodal points of the finite element mesh. In addition, the Kelvin transformation approach is sensitive to the fineness of discretization and a significant reduction in computed force error has been achieved by gradually increasing the mesh fineness. In contrast to the large errors associated to the simple truncation method and summarized in Table I, the error here has decreased from 0.47 to 0.08 per cent. There is one point worth noting for the accuracy of the related numerical field distributions. It is simply that force calculation methods often involve some form of numerical differentiation, either in terms of stored energies or potential distributions, and the errors inherent in potential distributions are enhanced when calculating forces numerically. It follows that the related distributions of magnetic flux densities and potential distributions are even more precise than the numerically computed forces. This precision level is completely satisfactory for most practical applications [26], especially when the approximations inherent in two-dimensional analysis are taken into account.

CONCLUSIONS

A discussion on two open boundary techniques used in conjunction with the finite element method is presented in this paper. To investigate the performance of the techniques of simple truncation and Kelvin transformation, a problem of force calculation has been chosen. The force to be evaluated is originated by currents in opposing directions in a set of two parallel and long conductive busbars placed in air. The analytical value of the force has been used to check the accuracy of numerical force computations.

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The discussion places emphasis on the numerical models developed to analyze the techniques computationally. When using simple truncation, the accuracy of numerical solutions depends on the position of the outer boundary. Very often, the mesh can be relatively coarse in remote locations, except very close to the region of prime interest, so a careful planning of graded meshes can reduce the modeling effort and computing cost. Numerical results by this technique emphasize the effect of increasing the size of the outer boundary on the number of nodes of the finite element meshes, values of computed forces and the corresponding numerical errors. The Kelvin transformation technique permits a significant reduction in the radial extent of the discretized domain; the accuracy of numerical solutions is easily enhanced by employing a gradual increase in the fineness of the finite element mesh.

The method of simple truncation can be used to advantage when introducing the concept of exterior problems in introductory courses on electromagnetics education. The use of different rules for locating the outer boundary and simulations using the average of solutions employed by the related technique known as strategic dual image can bring considerable insight into the subject. As far as high accuracy is concerned, the method is computationally expensive and inaccurate.

The Kelvin transformation method is a truncation-free approach, presents a strong correlation between accuracy and mesh fineness, and this is one of the strengths of the method. The method is easy to understand and can be readily implemented in most existing finite element CAD systems.

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