

Analysis of Finite Element formulations for computing electromagnetic fields in the human body

L. Bernard, L. Nicolas, L. Krähenbühl,

Université de Lyon - CNRS UMR5005 (Ampère), Ecole Centrale de Lyon, 69134 Ecully, France

N. Burais

Université de Lyon - CNRS UMR5005 (Ampère), Université Lyon 1, 69622 Villeurbanne, France

J.A. de Vasconcelos

Universidade Federal de Minas Gerais, Belo Horizonte, Brasil

Abstract— Due to the very specific electromagnetic properties of biological tissues, electromagnetic field calculation in the human body is difficult and the choice of the best numerical formulation is not straightforward. This question is analysed by solving a canonical problem: the scattering of a plane wave by concentric spheres. Electromagnetic properties of various biological tissues are considered to take into account the heterogeneity of the human body. Several finite element formulations are compared in the 10Hz-1GHz frequency range with respect to the analytical solution.

Index Terms— Biological effects of electromagnetic radiation, finite element methods, frequency domain analysis, nonhomogeneous media.

I. INTRODUCTION

Calculating induced electromagnetic fields inside the human body requires efficient numerical methods and is actually a challenging problem due to [1]:

- the complex structure of the human body (at the microscopic and the macroscopic scales)
- the unusual electromagnetic properties of the tissues of the human body,
- the great variety of sources which leads to electric and magnetic field exposure in a large range of frequencies (from 50Hz with power transmission lines to a few GHz with mobile phones).

Conventional electromagnetic numerical methods are usually applied to this kind of study. They are applied to the Maxwell's equations, with different assumptions depending on the frequency range. The efficiency of these methods and formulations is well-known when solving conventional problems with usual conductors or dielectrics. Since human exposure problems are very unusual, the accuracy of the computations must be evaluated. Such evaluation has already been performed when using classical methods, such as SPFD and FDTD methods, by studying a layered sphere problem in low frequency [2], [3]. But nothing has been done with the Finite Element Method (FEM) applied to the computation of electromagnetic quantities in the human body in a wide frequency range. In this paper, the FEM is used in the frequency domain between 10Hz and 1GHz, to solve Low Frequency (LF) and High Frequency (HF) formulations. The accuracy is evaluated with respect to the analytical solution of a 3D scattering problem: a layered sphere with the electric properties of biological tissues

scattering a plane electromagnetic wave.

Biological tissues are both lossy dielectric materials and ionic conductors. They are generally considered as non-magnetic and linear with respect to the electromagnetic fields in the case of low magnitude field exposure [4]. Although the characterization of biological tissues is very difficult, there are relevant publications providing a great amount of measurements [5]. It appears that the composition and microscopic structure of biological tissues result in very unusual values of the macroscopic electrical properties: the permittivity (ϵ) and the conductivity (σ) strongly depend on the frequency, and the permittivity has very high values in low frequency. It should be noticed also that these properties are very different from one tissue to another.

These specific properties of biological tissues make the problem of human exposure to electromagnetic fields an unusual one. First, the discretization must take into account the organ heterogeneity in the body: it implies that the studied domain has to be bounded in order to avoid computational overload. Then, considering the properties of biological tissues, the accuracy of conventional formulations must be evaluated with respect to the frequency. The *a priori* choice of the formulation based on the classical characteristic parameters of the problem is analysed and discussed considering an *a posteriori* evaluation of the computation based on efficiency parameters.

II. COMPUTATIONAL METHODS

The great amount of publications dealing with electromagnetic field computation in the human body shows that several numerical techniques may be applied.

The impedance method [6], [7] consists in discretizing the computational domain with a 3D regular grid. A value of impedance is set to each edge of the grid so that electric currents and potentials can be computed from the source of magnetic or electric field. The more general Finite Difference Method (FDM) [8] may be used in time (FDTD) or in frequency domain. It consists in using a 3D regular grid to discretize Maxwell's equations on the whole computational domain.

Originally, these methods use a very simple mesh on which the properties of each organ can easily be defined. This approach results in two difficulties:

- the mesh density may be excessively high in large organs and in the surrounding air, or,
- the staircases on the interfaces between organs may lead to local errors [3].

Finite difference methods that eliminate these problems exist and are now well developed even if they are more complicated to handle. A powerful alternative which avoids such difficulties [9] is the FEM which naturally works with unstructured meshes. In this paper, computations are performed using the GetDP [10] environment for assembling the FE matrices and Matlab to analyse and solve the linear system.

III. STUDIED PROBLEM AND FORMULATIONS

A. Studied problem

Defining a representative problem which has an analytical solution is required in order to evaluate the accuracy of the FEM and the associated formulations when computing electromagnetic fields in the human body. The studied problem is the scattering of a plane wave by a layered sphere (Fig. 1) [11].

Sources of electromagnetic fields are of various types resulting in different human exposure configurations. In low frequency, the exposure to electric fields and the exposure to magnetic fields can be studied separately but both of them may induce a significant electric current in the human body [12]. In high frequency, the electric and the magnetic fields are necessarily coupled. For the studied canonical problem, the source is a plane electromagnetic wave such that the incident magnetic and electric fields are: $H_i=1\text{A/m}$ and $E_i=377\text{V/m}$.

The size of the sphere is representative of the human head (20 centimeter diameter). In order to represent the heterogeneity at the organ scale, the sphere consists of 4 layers having the electrical properties of different biological tissues: bone, muscle, fat and blood. Dielectric properties are found in [5].

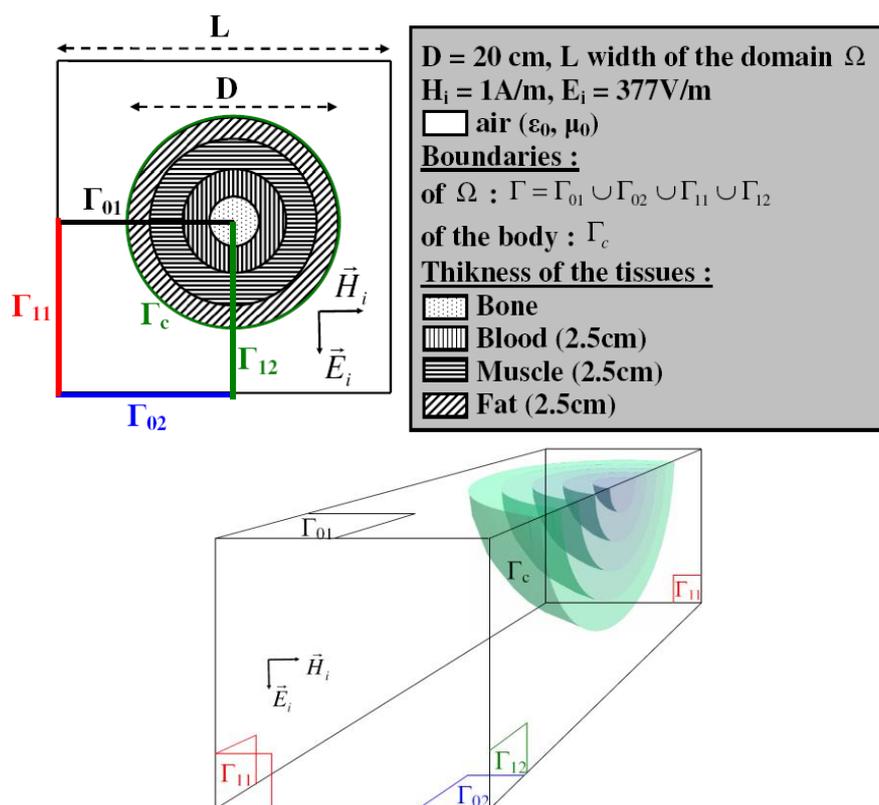


Fig. 1. Layered sphere plane wave scattering.

Although the reference problem is a free space one, the FEM requires a finite computational domain. Boundaries are chosen sufficiently far from the sphere in such a way that the incident field is not significantly changed when there is no propagation phenomenon.

B. Low frequency formulation (LF)

The studied LF formulation is the one proposed and validated by Bossavit in [12]. It consists in using the classical $\phi - \vec{A}$ formulation with an additional source term which takes into account the displacement current entering through the boundary (Γ_c) of the body. The computational domain is reduced to the volume of the body (Ω_c). In general, three steps are needed to solve the problem:

- compute the source magnetic vector \vec{A}_i (using Biot-Savart's law for air coils as example),
- compute the displacement current ($-j\omega\epsilon\vec{\nabla}V$) on the boundary of the body (considering the body as a perfect conductor),
- solve the $\phi - \vec{A}$ formulation with the two sources.

The weak form of the problem is then:

$$\int_{\Omega_c} (\sigma + j\omega\epsilon) \vec{\nabla} \phi \vec{\nabla} \phi' d\Omega = - \int_{\Omega_c} (\sigma + j\omega\epsilon) j\omega \vec{A}_i \vec{\nabla} \phi' d\Omega + \int_{\Gamma_c} j\omega\epsilon \vec{\nabla} V \phi' d\Gamma \quad (1)$$

where ϕ' is the appropriate scalar test function.

Theoretically, this formulation is useful whenever $\lambda/D \gg 1$ and $|\sigma + j\omega\epsilon| \gg \omega\epsilon_0$. When the first condition is satisfied, the plane wave of the problem studied here is assumed to create uniform electric and magnetic fields sources. The magnetic vector potential is directly obtained from the uniform magnetic field source. The displacement current on the surface of the body is obtained from a FE computation using the electro-quasi-static formulation in the surrounding air region with appropriate Dirichlet boundary conditions (this computation is named LF0 in the following). The LF formulation is also considered neglecting the displacement currents with respect to the conduction current (LF $\sigma \gg \omega\epsilon$).

C. High frequency formulations (HF)

The wave equation is considered separating the incident and the scattered electric fields (respectively \vec{E}_i and \vec{E}_s) [13]. Since the body has a magnetic permeability equal to the one of the vacuum, the weak form of the problem is:

$$\begin{aligned} & \int_{\Omega} \mu_0^{-1} \vec{\nabla} \times \vec{E}_s \cdot \vec{\nabla} \times \vec{E}' d\Omega + j\omega \int_{\Omega} (\sigma + j\omega\epsilon) \vec{E}_s \cdot \vec{E}' d\Omega \\ & = -j\omega \int_{\Omega_c} (\sigma + j\omega(\epsilon - \epsilon_0)) \vec{E}_i \cdot \vec{E}' d\Omega - \int_{\Gamma} \mu_0^{-1} \vec{n} \times \vec{\nabla} \times \vec{E}_s \cdot \vec{E}' d\Gamma \end{aligned} \quad (2)$$

The source is imposed by the value of the incident electric field in the body (first right-hand term). Various conditions are considered in order to truncate the computational domain:

1) Homogeneous Dirichlet Condition (HFDir0): This condition consists in imposing the scattered field to be null on the boundaries.

2) First Order Absorbing Boundary Condition (HFAB1): This condition assumes that the scattered field results in a plane wave which is normally incident to the boundaries ($\vec{n} \times \vec{\nabla} \times \vec{E}_s = -j\omega\epsilon_0^{-1}\vec{E}_s$).

3) Perfectly Matched Layers (HFPML): The considered PML has a thickness of 0.2m. A homogeneous Dirichlet condition is applied on the exterior boundary of the PML.

Transparent condition which uses integral equations coupled to the FEM has already been studied on a 2D case. The high computation cost induced by this condition is not affordable in the 3D problem studied here.

D. Frequency scaling (FS)

The HF formulation may lead to solve very badly conditioned linear systems when $\lambda/D \gg 1$. A technique called “frequency scaling” [2] can be used to avoid the problem. The electric field \vec{E}' is first computed with the HF formulation at pulsation $\omega' \neq \omega$ with $\sigma(\omega)$ and $\epsilon(\omega)$ (such that $\sigma(\omega) \gg \omega' \epsilon(\omega)$). Then, the electric field \vec{E} at pulsation ω is calculated from the relation:

$$\vec{E} = \frac{\omega}{\omega'} \vec{E}' \quad (3)$$

In the study presented here, the HF formulation is solved at 100 kHz and then scaled for any frequency in the 10Hz-1GHz frequency range.

IV. A PRIORI CHOICE OF THE FORMULATION AND A POSTERIORI EFFICIENCY EVALUATION

A. A priori choice of the formulation

The choice of the formulation depends on two main parameters: the ratio between the wavelength (λ) and the characteristic size (D) of the body, and the ratio between conduction and displacement currents ($\sigma/\omega\epsilon$). For human exposure problems in general, and for the specific problem studied here, the analysis of these parameters do not provide an obvious choice of the formulation in the whole frequency range. Fig. 2 shows the minimal value of the $\sigma/\omega\epsilon$ and λ/D and the corresponding tissue in function of the frequency. Considering the values of λ/D , propagation phenomena are expected to occur firstly in blood regions and for frequencies above 2MHz. Due to the very high values of permittivity in low frequency, it is difficult to know whether displacement currents have to be taken into account or not. They may not be negligible depending on the frequency and the tissue under consideration.

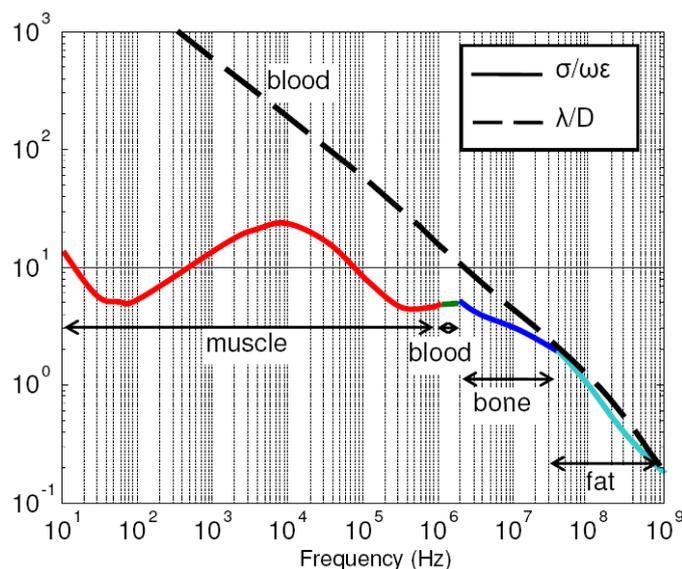


Fig. 2. Parameters for the *a priori* choice of the formulation.

B. A posteriori efficiency of the computation

The FEM leads to solve linear systems of the form $Ax = B$. The complexity of a FE model is determined by the complexity of the A matrix. The main parameters that characterize this complexity are:

- the number of columns,
- the density,
- the conditioning number.

Another criterion of efficiency is the relative difference between the FE solution and the analytical solution of the free space problem:

$$e = \frac{\left\| \vec{E} \right\| - \left\| \vec{E}_a \right\|}{\left\| \vec{E}_a \right\|} \quad (3)$$

where \vec{E} is the electric field computed with the FE method and \vec{E}_a is the analytical solution. The mean value of the relative error is evaluated on a large number of points inside the body.

V. RESULTS

All formulations are computed on the same tetrahedral mesh. Second order nodal elements are used for the LF formulations. Second order edge elements are used for the HF formulations.

The relative error with respect to the analytical solution (Fig. 3) can be analyzed on various frequency regions:

1) Below 100 kHz, HF formulations should not be used directly. These formulations lead to a very badly conditioned system (Fig. 4) and can not give accurate results. The frequency scaling technique (FS) may be used to avoid the problem but the LF formulation gives a better solution in terms of matrix conditioning, size and density (Table 1).

2) Between 100 kHz and 3 MHz, propagating phenomena and displacement current can be neglected. All the studied formulations give accurate results but the LF formulation is much more efficient than the others.

3) Between 3 and 10 MHz, propagation can still be neglected but displacement currents must be taken into account. The LF formulation remains the more efficient one.

4) Between 10 MHz and 200 MHz, propagation must be taken into account. Above 100 MHz, HFDir0 is not accurate because the scattered field is not negligible at the domain boundaries. The computational domain must be truncated using an appropriate technique, for example using PML (the accuracy of the HFAB1 formulation is the same the one of HFPML on this frequency range and is not presented here).

4) Above 200 MHz, the chosen mesh is not dense enough to obtain accurate results.

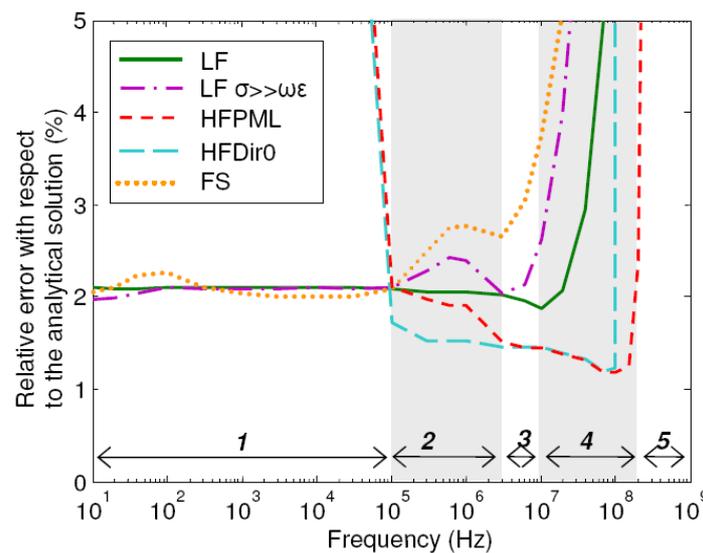


Fig. 3. Relative error with respect to the analytical solution for the different formulations.

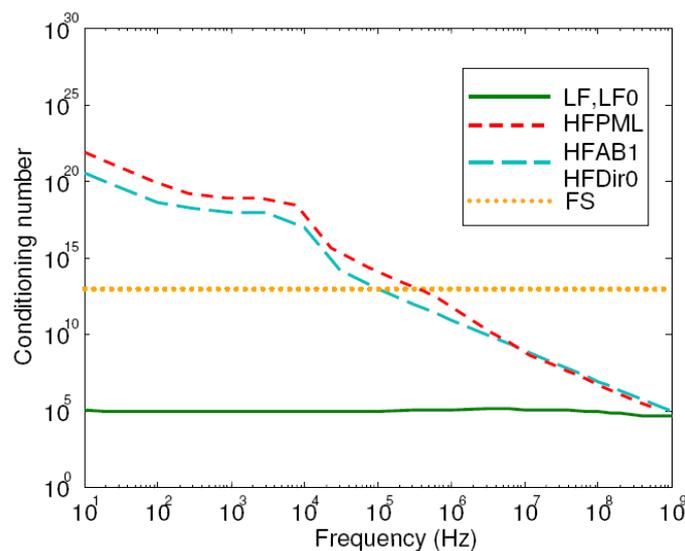


Fig. 4. Conditioning number as a function of the frequency for the different formulations.

TABLE I. LINEAR SYSTEM CHARACTERISTICS

Formulation	Number of columns of A	Density of A
LF0	11359	0.1736
LF	4676	0.5109
HFD _{ir} 0, HFPML, FS	24774	0.1211
HFAB1	26664	0.1127

VI. CONCLUSION

The specific properties of biological tissues make the problem of interaction between electromagnetic fields and human body to be an unusual one. In this paper, the efficiencies of several models based on LF and HF formulations are compared when solving a representative 3D problem. It is shown that on a large frequency range, from 10 Hz to 10 MHz, a LF formulation taking into account the displacement currents gives accurate results and is computationally much more efficient than the FS formulation. Above 10 MHz, HF formulation must be used to take into account propagation phenomena in the body. Above 100 MHz, an efficient method must be used in order to simulate the free-space at the boundary of the computational domain. Due to the specific electrical properties of the body tissues and their heterogeneity, the choice of the formulation based on a *posteriori* efficiency evaluation is different from the choice based on classical *a priori* criteria.

The results presented here depend on the size of the studied body. A possible enhancement of this work would be to consider a more realistic model of the whole human body.

REFERENCES

- [1] N Siauve, R Scorretti, N Burais, L Nicolas, A Nicolas, "Electromagnetic fields and human body: a new challenge for the electromagnetic field computation", *Compel, The Int. Jour. for Comp. and Math. in Electrical and Electronic Eng.*, vol 22, n°3, 2003, pp. 457-469.
- [2] Om P. Gandhi, Jin-Yan Chen, "Numerical dosimetry at power-line frequencies using anatomically based models", *Bioelectromagnetics Supplement*, 1:43-60 (1992).
- [3] M. A. Stuchly, T. W. Dawson, "Interaction of low-frequency electric and magnetic fields with the human body", *IEEE Proceedings*, vol. 88, NO. 5, May 2000.
- [4] S Grimnes, O G Martinsen, Bioimpedance and bioelectricity basics, *Academic Press*, 2000.
- [5] S.Gabriel et al., "The dielectric properties of biological tissues: II. Measurements in the frequency range 10 Hz to 20 GHz", *Phys. Med. Biol.* 41 2251-2269, 1996.
- [6] Nadeem M., Thorlin T., Gandhi O.P., Persson M., "Computation of Electric and Magnetic Stimulation in Human Head Using the 3-D Impedance Method", *IEEE Transaction On Biomedical Engineering*, vol. 50, No. 7, July 2003.
- [7] Dawson T.W., Stuchly M.A., "Effects of skeletal muscle anisotropy on human organ dosimetry under 60Hz uniform magnetic field exposure", *Phys. Med. Biol.* 43 (1998) 1059-1074.
- [8] C. M. Furse and O. P. Gandhi, "Calculation of Electric Fields and Currents Induced in a Millimeter-Resolution Human Model at 60 Hz Using the FDTD Method", *Bioelectromagnetics* 19:293-299 (1998).
- [9] Scorretti R., Burais N., Fabregue O., Nicolas A., Nicolas L., "Computation of the induced current density into the human body due to LF magnetic field generated by realistic devices", *IEEE Transactions on Magnetics*, vol. 40, n° 2, pp. 643-646, 2004.
- [10] P. Dular, C. Geuzaine, GetDP Reference Manual, Edition 4.1, February 2005.
- [11] D.W. Mackowski, R.A. Altenkirch, M.P. Menguc, "Internal absorption cross sections in a stratified sphere", *Appl. Opt.* 29(10):15551-15559, 1990.
- [12] A. Bossavit, "Mathematical modeling of the problem of micro-currents generated in living bodies by power lines", *Int. J. of Applied Electromagnetics in Materials*, 4 (1994) 291-299.
- [13] P.D. Ledger, "An hp-adaptive finite element procedure for electromagnetic scattering problems", PhD thesis, Dept. Civil Engineering, University of Wales, Swansea, 2002.